

Mechanically Persistent Oscillator Supplied With Ramp Signal

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ABSTRACT: This paper submits a new tactic known as the integral Rohit transform (RT) for uncovering the response of a mechanically persistent oscillator supplied with a ramp signal. It let outs that RT is an operative tool for uncovering the response of a mechanically persistent oscillator supplied with a ramp signal.

Keywords: RT, mechanically persistent oscillator, Ramp Signal

1. INTRODUCTION

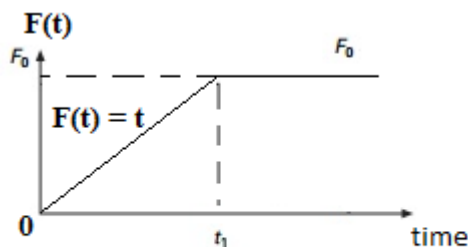


Figure: Ramp Function

The ramp signal (shown in the figure above) is explicated as:

$$F(t) = t \text{ for } 0 < t < t_1$$

$$= F_0 \text{ for } t \geq t_1.$$

The author Rohit Gupta has proffered the integral RT in recent years [1, 2].

The RT is explicated as $R\{g(t)\} = G(q) = q^3 \int_0^{\infty} e^{-qt} g(t) dt$. Here $t \geq 0$ and the integral is merging.

A unit step function [3] is explicated as $U(t - a) = 0$ for $t < a$ and 1 for $t \geq a$.

The RT of a unit step function is stated as

$$R\{U(t - c)\} = q^3 \int_0^{\infty} e^{-qt} U(t - c) dt$$

$$R\{U(t - c)\} = q^3 \int_c^{\infty} e^{-qt} dt$$

$$R\{U(t - c)\} = q^2 e^{-qc}$$

Shifting property of RT:

If $R\{g(t)\} = G(q)$, then $R\{g(t - c)U(t - c)\} = e^{-qc}G(q)$.

Proof:

$$R\{g(t - c)U(t - c)\} = q^3 \int_0^\infty e^{-qt} g(t - c)U(t - c) dt$$

$$R\{g(t - c)U(t - c)\} = q^3 \int_c^\infty e^{-qt} g(t - c) dt$$

$$R\{g(t - c)U(t - c)\} = q^3 \int_0^\infty e^{-q(v+c)} g(v) dv, \text{ where } v = t - c$$

$$R\{g(t - c)U(t - c)\} = e^{-q(c)} q^3 \int_0^\infty e^{-qv} g(v) dv$$

$$R\{g(t - c)U(t - c)\} = e^{-q(c)} q^3 \int_0^\infty e^{-qt} g(t) dt$$

$$R\{g(t - c)U(t - c)\} = e^{-q(c)} G(q)$$

The RT of some basic functions is stated as

- ❖ $R\{t^n\} = \frac{n!}{q^{n+2}}$, where n is $0, 1, 2, \dots$
- ❖ $R\{\text{sinct}\} = \frac{c q^3}{q^2 + c^2}$,
- ❖ $R\{\text{cosct}\} = \frac{q^4}{q^2 + c^2}$,

The RT of some derivatives is explicated as

$$R\{g'(t)\} = qR\{g(t)\} - q^3 g(0),$$

$$R\{g''(t)\} = q^2 R\{g(t)\} - q^4 g(0) - q^3 g'(0), \text{ and so on.}$$

2. METHODOLOGY

A mechanically persistent oscillator [4, 5] supplied with ramp signal is specified by the following equation:
 $m\ddot{y}(t) + ky(t) = F(t)$

Or

$$\ddot{y}(t) + \omega_0^2 y(t) = \frac{F(t)}{m} \dots (1)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$, $F(t)$ is a ramp signal, $y(0) = 0$ and $\dot{y}(0) = 0$.

The RT of (1) provides

$$q^2 \bar{y}(q) - q^4 y(0) - q^3 \dot{y}(0) + \omega_0^2 \bar{y}(q) = \frac{1}{m} q^3 \int_0^\infty e^{-qt} F(t) dt$$

$$\Rightarrow q^2 \bar{y}(q) - q^4 y(0) - q^3 \dot{y}(0) + \omega_0^2 \bar{y}(q) = \frac{1}{m} q^3 \int_0^{t_1} e^{-qt} t dt + \frac{F_0}{m} q^3 \int_{t_1}^\infty e^{-qt} dt$$

Here $\bar{y}(q)$ denotes the RT of $y(t)$.

Put $y(0) = 0$ and $\dot{y}(0) = 0$ [3], we get

$$q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{1}{m} q^3 \int_0^{t_1} e^{-qt} t dt + \frac{F_0}{m} q^3 \int_{t_1}^\infty e^{-qt} dt$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = -\frac{1}{m} q^2 [t_1 e^{-qt_1}] + \frac{1}{m} q^2 \int_0^{t_1} e^{-qt} dt - q^2 \frac{F_0}{m} [e^{-qt_1}]$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = -\frac{1}{m} q^2 [t_1 e^{-qt_1}] - \frac{1}{m} q [e^{-qt_1} - 1] - q^2 \frac{F_0}{m} [e^{-qt_1}]$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = -\frac{1}{m} \{q^2 t_1 e^{-qt_1} - q e^{-qt_1} + q - q^2 F_0 e^{-qt_1}\}$$

$$\Rightarrow q^2 \bar{y}(q) + \omega_0^2 \bar{y}(q) = \frac{1}{m} \{q^2 F_0 e^{-qt_1} + q e^{-qt_1} - q - q^2 t_1 e^{-qt_1}\}$$

$$\begin{aligned} \Rightarrow \bar{y}(q) &= \frac{1}{m} \left\{ \frac{q^2}{(q^2 + \omega_0^2)} F_o e^{-qt_1} + \frac{q}{(q^2 + \omega_0^2)} e^{-qt_1} - \frac{q}{(q^2 + \omega_0^2)} - \frac{q^2 t_1 e^{-qt_1}}{(q^2 + \omega_0^2)} \right\} \\ \Rightarrow \bar{y}(q) &= \frac{1}{m} \left\{ \frac{q^4}{q^2(q^2 + \omega_0^2)} F_o e^{-qt_1} + \frac{q^3}{q^2(q^2 + \omega_0^2)} e^{-qt_1} - \frac{q^3}{q^2(q^2 + \omega_0^2)} \right. \\ &\quad \left. - \frac{q^4}{q^2(q^2 + \omega_0^2)} t_1 e^{-qt_1} \right\} \\ \Rightarrow \bar{y}(q) &= \frac{1}{m} \left\{ \frac{q^2}{(\omega_0^2)} F_o e^{-qt_1} - \frac{q^4}{(\omega_0^2)(q^2 + \omega_0^2)} F_o e^{-qt_1} + \frac{q}{(\omega_0^2)} e^{-qt_1} \right. \\ &\quad - \frac{q^3}{(\omega_0^2)(q^2 + \omega_0^2)} e^{-qt_1} - \frac{q}{(\omega_0^2)} + \frac{q^3}{(\omega_0^2)(q^2 + \omega_0^2)} - \frac{q^2}{(\omega_0^2)} t_1 e^{-qt_1} \\ &\quad \left. + \frac{q^4}{(\omega_0^2)(q^2 + \omega_0^2)} t_1 e^{-qt_1} \right\} \end{aligned}$$

Applying inverse RT, we get

$$\begin{aligned} y(t) &= \frac{1}{m} \left\{ \frac{F_o}{(\omega_0^2)} U(t - t_1) - \frac{F_o \cos \omega_0(t - t_1)}{(\omega_0^2)} U(t - t_1) + \frac{(t - t_1)}{(\omega_0^2)} U(t - t_1) \right. \\ &\quad - \frac{\sin \omega_0(t - t_1)}{\omega_0(\omega_0^2)} U(t - t_1) - \frac{(t)}{(\omega_0^2)} + \frac{\sin \omega_0(t)}{\omega_0(\omega_0^2)} - \frac{t_1}{(\omega_0^2)} U(t - t_1) \\ &\quad \left. + \frac{t_1 \cos \omega_0(t - t_1)}{(\omega_0^2)} U(t - t_1) \right\} \\ \Rightarrow y(t) &= \frac{1}{m(\omega_0^2)} \left\{ F_o U(t - t_1) - F_o \cos \omega_0(t - t_1) U(t - t_1) + (t - t_1) U(t - t_1) \right. \\ &\quad - \frac{\sin \omega_0(t - t_1)}{\omega_0} U(t - t_1) - t + \frac{\sin \omega_0(t)}{\omega_0} - t_1 U(t - t_1) \\ &\quad \left. + t_1 \cos \omega_0(t - t_1) U(t - t_1) \right\} \\ \Rightarrow y(t) &= \frac{1}{k} \left\{ F_o U(t - t_1) - F_o \cos \omega_0(t - t_1) U(t - t_1) + (t - t_1) U(t - t_1) \right. \\ &\quad - \frac{\sin \omega_0(t - t_1)}{\omega_0} U(t - t_1) - t + \frac{\sin \omega_0(t)}{\omega_0} - t_1 U(t - t_1) \\ &\quad \left. + t_1 \cos \omega_0(t - t_1) U(t - t_1) \right\} \end{aligned}$$

For $t < t_1$,

$$y(t) = \frac{1}{k} \left[-t + \frac{\sin \omega_0 t}{\omega_0} \right]$$

Or

$$y(t) = \frac{1}{k} \left[\frac{\sin \omega_0 t}{\omega_0} - t \right]$$

For $t > t_1$,

$$y(t) = \frac{1}{k} \left\{ F_o - F_o \cos \omega_0(t - t_1) - \frac{\sin \omega_0(t - t_1)}{\omega_0} + \frac{\sin \omega_0(t)}{\omega_0} - 2t_1 + t_1 \cos \omega_0(t - t_1) \right\}$$

Or

$$y(t) = \frac{1}{k} \left\{ F_o [1 - \cos \omega_0 (t - t_1)] - \frac{1}{\omega_0} [\sin \omega_0 (t - t_1) - \sin \omega_0 (t)] - t_1 [2 - t_1 \cos \omega_0 (t - t_1)] \right\}$$

3. CONCLUSION

This paper has typified the RT for uncovering the response of a mechanically persistent oscillator supplied with a ramp signal. A new tactic has been fruitfully drawn on for uncovering the response of a *mechanically* persistent oscillator supplied with a ramp signal.

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CONFLICTS OF INTEREST

The author declares no conflict of interest

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