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Solving Problems of Physical or Basic Sciences via Rohit Transform

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ABSTRACT: The problems such as evolution and withering of population, immersion of glucose by the body, escalation of epidemics and reversal of gross domestic product (GDP) over time, Law of heating or cooling given by Newton, radioactive disintegration law, and velocity of an object which is falling vertically downwards under the operation of gravity pull, in physical or basic sciences, are delineated by differential equations. These problems delineated by differential equations are generally solved by the calculus method which requires complicated computations. In the present article, an advanced integral transform-Rohit transform (RT) is conferred for solving the problems delineated by differential equations in physical or basic sciences. The results come by confirming that the RT is a simple and more effective scientific tool for dealing with such problems delineated by differential equations in physical or basic sciences than the ordinary calculus method.

Keywords: Rohit transform (RT), physical or basic sciences, differential equations

1. INTRODUCTION

Generally, the problems in the physical or basic sciences such as evolution and withering of population, absorption of glucose by the body, escalation of epidemics and reversal of gross domestic product (GDP) over time, Law of heating or cooling given by Newton, radioactive disintegration law, and velocity of a particle falling downwards freely under the operation of gravity, delineated by differential equations, are solved by calculus method which requires complicated computations [1]-[5]. This article illustrates the RT for solving the differential equations delineating the problems in physical or basic sciences. These problems are mostly found in Economics, Environmental Management, Biology, Chemistry, Physics, Engineering, etc. The RT has been offered by the author Rohit Gupta in recent years [6] and has been enforced for solving problems involving initial values delineated by differential equations in the physical or basic sciences [7]-[8].

The article is outlined as: First, a brief inception of the RT is laid out. Second, the enactment of the RT to real-life or physical or basic sciences problems is explained. Finally, the argumentation and the deduction are furnished.

The process of resolving the problems delineated by differential equations in physical or basic sciences by relating the RT consists of three foremost steps: a. the problems of the physical or basic science (delineated by differential equations) are transformed into algebraic equations, known as auxiliary equations. b. The auxiliary equations are worked out by a wholly algebraic scheme. c. The workout in ‘b’ is transfigured back by relating inverse RT, resulting in the solutions of physical or basic sciences problems.

The key inclination for relating RT to solve problems delineated by differential equations in physical or basic sciences is that the process of working out the differential equations delineating the problems in physical or basic sciences is simplified to an algebraic problem. This exercise of swapping the problem of calculus into an algebraic problem is known as operational calculus.

The RT has two main favors over the calculus method: i. Problems involving initial values are resolved without first confirming a universal or general solution. ii. A non-homogenous differential equation is resolved without first resolving the corresponding homogeneous differential coefficient (or equation).

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The RT swaps a function into another advanced function by making use of integration. The RT of a function $\mathrm{h(t)}$, t ≥ 0 is defined as $G(s) = s^3 \int_0^\infty e^{-st} h(t)dt$. Here, the given integral is convergent [6]-[8] and s is either complex or real. One calls RT an integral transform because it swaps a function in one space to a function in another space making use of integration that involves a function: k(s, t) = e^{-st}, known as the kernel. It is a function of two alterable i.e. s and t in the two spaces.

The RT of some basic functions is given as

- $R\{t^n\} = \frac{n!}{s^{n-2}}$, $where\ n = 0,1,2, \dots$
- $R\{e^{et}\} = \frac{s^3}{s-e}$, $s > e$, where e is some constant.
- $R\{sinet\} = \frac{e s^3}{s^2+e^2}$, $s > 0$
- $R\{coset\} = \frac{s^4}{s^2+e^2}$, $s > 0$

The RT of derivatives of $\mathrm{h(t)}$ is given as

$R\left\{\frac{dh(t)}{dt}\right\} = sG(s) - s^3h(0),$

$R\left\{\frac{d^2h(t)}{dt^2}\right\} = s^2G(s) - s^4h(0) - s^3h'(0),$

$\mathrm{R}\left\{\frac{d^3h(t)}{dt^3}\right\} = \mathrm{s^3G(s) - s^5h(0) - s^4h'(0) - s^3h''(0)},$

And so on.

2. METHODOLOGY

In this section, the RT is employed for solving problems such as evolution and withering of population, immersion of glucose by the body, escalation of epidemics and reversal of gross domestic product (GDP) over time, the law of heating or cooling given by Newton, radioactive disintegration law, and velocity of an object which is falling vertically downwards under the operation of gravity pull, delineated by differential equations in physical or basic sciences.

Problem 1: Growth and Decay of Population

In the Malthusian model, the growth and decay of the population at any instant t is given by the ensuing equation as [5]

$$\frac{dP(t)}{dt} = \lambda P(t) \dots \dots (1),$$

$where\ \lambda$ is any positive or negative constant. It is positive for the birth rate and negative for the death rate. $P(t)$ is the population at any instant t. The population at initial time t = 0 is P_0 i.e. $P(0) = P_0$.

Solution: Relating the RT to equation (1), we have

$s\,P(s) - s^3P(0) = \lambda P(s)$

Put $P(0) = P_0$ and reordering, we have

$$P(s) = \frac{P_0\,s^3}{s-\lambda}$$

Relating the inverse RT, we have

$P(t) = P_0e^{\lambda t}$

When t approaches infinity, $P(t)$ = 0 for negative values of λ i.e. population approaches 0 if death rate > birth rate, and $P(t)$ = infinite for positive values of λ i.e. population approaches ∞ if birth rate > death rate.

It is clear from the above equation that the population grows or decays exponentially.

In the Verhulst model, the growth and decay of population at any instant t is given by the ensuing equation as [5]

$$\frac{dP(t)}{dt} = \lambda P(t)[1 - \lambda P(t)] \dots \dots (2),$$

$where\ \lambda$ is any positive/negative constant and $P(t)$ is the population at any instant t. The population at initial time t = 0 is P_0 i.e. $P(0) = P_0$.

Solution: Equation (2) is rewritten as

$$\frac{dP}{dt} = \lambda P - \lambda^2 P^2$$

Or

$\frac{dP}{dt} - \lambda P = -\lambda^2 P^2$……. (2a)

Let Z = 1/P, then $-P^2 \frac{dZ(t)}{dt} = \frac{dP(t)}{dt}$, then equation (2a) becomes

$$-P^2 \frac{dZ(t)}{dt} - \lambda P = -\lambda^2 P^2$$

Or

$$\frac{dZ}{dt} + \lambda Z = \lambda^2$$

Or

$$\frac{dZ}{dt} = -\lambda Z + \lambda^2$$

Relating the RT, we have

$$s\, Z(s) - s^3 Z(0) = -\lambda Z(s) + \lambda^2 s^2$$

Or

$$s\, Z(s) + \lambda Z(s) = s^3 \frac{1}{P_0} + \lambda^2 s^2$$

Or

$$Z(s) = \frac{1}{P_0} \frac{s^3}{s+\lambda} + \frac{\lambda^2 s^2}{s+\lambda}$$

Or

$$Z(s) = \frac{1}{P_0} \frac{s^3}{s+\lambda} + \lambda[\, s^2 - \frac{s^3}{s+\lambda}\,]$$

Or

$$Z(s) = \frac{(\frac{1}{P_0} - \lambda)\, s^3}{s+\lambda} + \lambda\, s^2$$

Relating the inverse RT, we have

$$Z(t) = (\frac{1}{P_0} - \lambda)e^{-\lambda t} + \lambda$$

Therefore,

$$P(t) = \frac{1}{(\frac{1}{P_0} - \lambda)e^{-\lambda t} + \lambda}$$

Or

$$P(t) = \frac{P_0}{(1 - P_0\lambda)e^{-\lambda t} + P_0\lambda}$$

When t approaches infinity, $P(t) = \frac{1}{\lambda}$ which is independent of initial population P_0.

Problem 2: Glucose immersion by the body

Suppose $G(t)$ is the units of glucose in the bloodstream at t > 0. Since the glucose being immersed by the body is leaving the bloodstream, therefore, $G(t)$ satisfies the ensuing equation as [5], [9]

$$\frac{dG(t)}{dt} = -\lambda G(t) \;\dots\dots (3),$$

$where\ \lambda$ is any positive constant. The units of glucose in the bloodstream at initial time t = 0 is G_0 i.e. $G(0) = G_0$.

Solution: Relating the RT to equation (3), we have

$$s\, G(s) - s^3 G(0) = -\lambda G(s)$$

Put $G(0) = G_0$ and reordering, we have

$G(s) = \frac{G_0\, s^3}{s+\lambda}$

Relating the inverse RT, we have

$G(t) = G_0 e^{-\lambda t}$

It is clear from the above equation that the units of glucose in the bloodstream fall off exponentially.

Now, if the glucose is injected at a fixed rate of r units/sec, then $G(t)$ satisfies the ensuing equation as [5]:

$$\frac{dG(t)}{dt} = -\lambda G(t) + r \ldots\ldots (3a)$$

Here $-\lambda G(t)$ is due to the immersion of the glucose by the body and r is due to the injection. Also, we assume $G(t) = G_0$.

Solution: Relating the RT to equation (3a), we have

$s\,G(s) - s^3 G(0) = -\lambda G(s) + r\, s^2$

Put $G(0) = G_0$ and reordering, we have

$G(s) = \frac{G_0\, s^3}{s+\lambda} + \frac{r\, s^2}{s+\lambda}$

Or

$G(s) = \frac{G_0\, s^3}{s+\lambda} + \frac{r\, s^2}{s+\lambda}$

Or

$G(s) = \frac{G_0\, s^3}{s+\lambda} + \frac{r}{\lambda}[\, s^2 - \frac{s^3}{s+\lambda}]$

Or

$G(s) = \frac{(G_0 - \frac{r}{\lambda})\, s^3}{s+\lambda} + \frac{r}{\lambda}\, s^2$

Relating the inverse RT, we have

$G(t) = (G_0 - \frac{r}{\lambda})\, e^{-\lambda t} + \frac{r}{\lambda}$

Problem 3: Spread of epidemics

The Spread of the epidemics model is delineated by the ensuing equation as [5], [10]

$$\frac{dI(t)}{dt} = \lambda I(t)[N - I(t)] \ldots\ldots..(4)$$

$where\ \lambda$ is any positive constant and $I(t)$ is the population of infected people at any time t, N is the total population of susceptible people and $N - I(t)$ is the population of susceptible people, but still not infected. The population of infected people at the initial time t = 0 is I_0 i.e. $I(0) = I_0$.

Solution: Equation (4) is rewritten as

$\frac{dI}{dt} = N\lambda I - \lambda\, I^2$

Or

$\frac{dI}{dt} - N\lambda I = -\lambda\, I^2$……. (4a)

Let Z = 1/I, then $-\, I^2 \frac{dZ(t)}{dt} = \frac{dI(t)}{dt}$, then equation (4a) becomes

$-\, I^2 \frac{dZ(t)}{dt} - N\lambda I = -\lambda\, I^2$

Or

$$\frac{dZ}{dt} + N\lambda Z = \lambda$$
Or
$$\frac{dZ}{dt} = -N\lambda Z + \lambda$$
Relating the RT, we have
$$s\,Z(s) - s^3 Z(0) = -N\lambda Z(s) + \lambda\, s^2$$
Or
$$s\,Z(s) + N\lambda Z(s) = s^3 \frac{1}{I_0} + \lambda\, s^2$$
Or
$$Z(s) = \frac{1}{I_0}\frac{s^3}{s+N\lambda} + \frac{\lambda\, s^2}{s+N\lambda}$$
Or
$$Z(s) = \frac{1}{I_0}\frac{s^3}{s+N\lambda} + \frac{1}{N}[\, s^2 - \frac{s^3}{s+N\lambda}\,]$$
Or
$$Z(s) = \frac{(\frac{1}{I_0} - \frac{1}{N})\, s^3}{s+N\lambda} + \frac{1}{N}\, s^2$$
Relating the inverse RT, we have
$$Z(t) = (\frac{1}{I_0} - \frac{1}{N})e^{-N\lambda t} + \frac{1}{N}$$
Therefore,
$$I(t) = \frac{1}{(\frac{1}{I_0} - \frac{1}{N})e^{-N\lambda t} + \frac{1}{N}}$$
Or
$$I(t) = \frac{NI_0}{(N - I_0)e^{-N\lambda t} + I_0}$$
When t approaches infinity, $I(t) = N$ which means that all the susceptible people eventually become infected.

Problem 4: Changes in GDP

The changes in GDP w.r.t time are directly proportional to the current GDP. The ensuing equation describes the state x of the GDP of the economy [5], [11], [12]
$$\frac{dx(t)}{dt} = \lambda x(t) \dots\dots\dots (5)$$
where λ is any positive/negative constant and $x(t)$ is the GDP at any time t. The GDP at initial time t = 0 is x_0 i.e. $x(0) = x_0$.

Solution: Relating the RT to equation (5), we have
$$s\,X(s) - s^3 x(0) = \lambda X(s)$$
Put $x(0) = x_0$ and reordering, we have
$$X(s) = \frac{x_0\, s^3}{s - \lambda}$$
Relating the inverse RT, we have
$$x(t) = x_0 e^{\lambda t}$$
When t approaches infinity, x = 0 for negative values of λ and x = infinite for positive values of λ.
It is clear from the above equation that the GDP falls off or shoots up exponentially.

Problem 5: Motion of a particle falls vertically downwards under the operation of gravity pull in a viscous medium

The motion of a particle falls vertically under gravity and experiences a force of air resistance is given by [13], [14]

$$\frac{dv(t)}{dt} = -\lambda v(t) + g \dots\dots(5)$$

Here $-\lambda v(t)$ is due to the force of air resistance and g is the acceleration due to gravity. Also, we assume $v(0) = 0$.

Solution: Relating the RT to equation (5), we have

$$s\,V(s) - s^3 v(0) = -\lambda V(s) + g\,s^2$$

Put $v(0) = 0$ and reordering, we have

$$V(s) = \frac{g\,s^2}{s+\lambda}$$

Or

$$V(s) = \frac{g}{\lambda}[\,s^2 - \frac{s^3}{s+\lambda}]$$

Or

$$V(s) = -\frac{g}{\lambda}\frac{s^3}{s+\lambda} + \frac{g}{\lambda}\,s^2$$

Relating the inverse RT, we have

$$v(t) = -\frac{g}{\lambda}\,e^{-\lambda t} + \frac{g}{\lambda}$$

Or

$$v(t) = \frac{g}{\lambda}(1 - e^{-\lambda t})$$

When t approaches infinity, v = $\frac{g}{\lambda}$ which is equal to the maximum velocity of the particle falling vertically under gravity.

Problem 6: Law of heating or cooling

The law of heating or cooling given by Newton is delineated by the ensuing equation as [1], [14]

$$\frac{dT(t)}{dt} = -\lambda(T(t) - T_0) \dots\dots(6)$$

Here T_0 is the fixed temperature of the surrounding medium and T is the temperature at any instant t. Also, we assume $T(0) = T_1$

Solution: Relating the RT to equation (6), we have

$$s\,T(s) - s^3 T(0) = -\lambda T(s) + \lambda T_0\,s^2$$

Put $T(0) = T_1$ and reordering, we have

$$T(s) = \frac{T_1\,s^3}{s+\lambda} + \frac{\lambda T_0\,s^2}{s+\lambda}$$

Or

$$T(s) = \frac{T_1\,s^3}{s+\lambda} + T_0[\,s^2 - \frac{s^3}{s+\lambda}]$$

Or

$$T(s) = \frac{(T_1 - T_0)\,s^3}{s+\lambda} + T_0\,s^2$$

Relating the inverse RT, we have

$$T(t) = (T_1 - T_0)e^{-\lambda t} + T_0$$

Or

$$T(t) = (T_1 - T_0)e^{-\lambda t} + T_0$$

It is clear from the above equation that the temperature of the body falls off exponentially.

Problem 7: Radioactive disintegration law

The Radioactive disintegration law is represented by the ensuing equation as [1]

$$\frac{dN(t)}{dt} = -\lambda N(t) \dots\dots (7)$$

Here λ is a positive constant. $N(t)$ is the atoms present in the element at any instant t and $N(0) = N_0$ is the atoms present in the element at t = 0.

Solution: Relating the RT to equation (7), we have

$$s\,N(s) - s^3 N(0) = -\lambda N(s)$$

Put $N(0) = N_0$ and reordering, we have

$$N(s) = \frac{N_0\, s^3}{s + \lambda}$$

Relating the inverse RT, we have

$$N(t) = N_0 e^{-\lambda t}$$

It is confirmed from this equation that the atoms in the element fall off exponentially.

3. DISCUSSION

The RT has been put in place flourishingly for solving the problems delineated by differential equations such as evolution and withering of population, immersion of glucose by the body, escalation of epidemics, reversal of GDP over time, Law of heating or cooling given by Newton, radioactive disintegration law, and velocity of an object which is falling vertically downwards under the operation of gravity pull, in physical or basic sciences. The solutions that come by relating the RT are identical to those that come by relating the calculus method, demonstrating that the RT is a simpler and more effective scientific device for dealing with problems in physical or basic sciences than the calculus method.

4. CONCLUSION

The RT is concluded to be a simple and effective scientific device for solving the problems delineated by differential equations such as evolution and withering of population, absorption of glucose by the body, escalation of epidemics, reversal of GDP over time, Law of heating or cooling given by Newton, radioactive disintegration law, and velocity of an object which is falling vertically downwards under the operation of gravity pull, and others in physical or basic sciences. It is found that the population shoots up or falls off exponentially. The population approaches zero if the death rate exceeds the birth rate, and the population approaches infinity if the birth rate exceeds the death rate. The units of glucose in the bloodstream fall off exponentially. All the susceptible people eventually become infected after a long period of time. The GDP of the economy falls off or shoots up exponentially. The velocity of an object which is falling vertically downwards under the operation of gravity pull in a viscous medium shoots up exponentially, and the value g/λ is the maximum velocity of the object which is falling vertically downwards under the operation of gravity pull in an air medium. The temperature of the body falls off exponentially. The atoms in the sample fall off exponentially. These results validate the RT as an effective scientific tool for solving problems in physical or basic sciences compared to the ordinary calculus method.

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The authors declare no conflict of interest

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