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Studying the Structural behavior of Walls of Fluid Basins Using the Finite Difference Method

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ABSTRACT: The real concern, which represents the dangerous places in the tanks containing liquid materials, is those where cracks occurs in the walls of those tanks. To determine the zone areas where the maximum precipitation occurs, a study of those walls had to be conducted and the ideal dimensions of those walls had to be searched for in order to control the highest precipitation and thus reduced the possibility of cracks occurring as a result of the large precipitation of the walls and thus avoid being exposed to the risk of the liquids containing in those tanks penetrating into the walls and thus the possibility of the collapse of those and exposure to the loss of liquids and the economic losses and environmental risks that it causes. This paper deals with a new derivation to prediction the elastic behavior of tank wall supported with fixed ended on three edges and freely at the fourth and subjected to hydrostatic liquid triangular distributed load using central finite difference method. The new model was verified by compares with the classical model obtained by Timoshenko and Woinowsky-Krieger based on theory of plates and shells with a good agreement. The present parametric study deals with tank wall with length to depth ratios of 1, 2, 3 and 4 were considered. The results obtained from the analysis showed that the maximum deflection increased by 10.3%, 23.7% and 32.5% for L/h=2, 3, and 4 compares with L/h=1.

Keywords: finite difference method, liquid tanks wall, Deflection, Elastic, Dangerous

1. INTRODUCTION

The analysis of liquid tank depends on the locations of the tank, i.e underground, overhead and on ground liquid tank. The tanks can be made in different shapes usually rectangular and circular shapes are mostly used but may be in other irregular shapes. The tanks can be classified depending on material which made from it like steel, polypropylene or reinforced concrete and etc. The design of liquid tank should be based on the sufficient resistance to cracking and avoided permissible maximum deflection due to different kinds of static and seismic or dynamic effects. This research proposed a mathematical model for computing the lateral deflection in rectangular liquid tank walls based on the plate analogy. Likes this works is necessary to give the engineers an obvious idea to understand the elastic structural behavior of the supporting wall under the effect of transvers liquid loads.

Timoshenko and Krieger [1] proposed a method for calculating the maximum wall precipitation at free end according to plates and shells theory. Further, a finite element method is also another method can be used for calculating the maximum deflection at the free end of tank wall. In this research, the reliance on the technology of the central finite difference will be used to solve the fourth order partial differential equation and transform it to number of first order linear equations derived to calculate the elastic deflections at all nodes of the walls that supporting the liquid loads the full walls transvers area. There are four cases of these tank walls were studied in parametric study: First case was the ratio between the height and length of wall was 1.0 ((L/h=1), the second case was a wall with horizontal length equal to twice the vertical height (L/h=2), the third case was a wall with horizontal length equal to three times the vertical height (L/h=3) and the fourth case was a wall with horizontal length equal to four times of the vertical height (L/h=4). The walls were bounded as restrained (fixed) or clamped on two opposite vertical edges and lower horizontal edge, but free on the upper horizontal edge.

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The analysis using finite difference method (FDM) applied to the tank wall subjected to hydraulic fluid pressure distributed as a triangular along the wall height. The loads starts with intensity (ɤh) at the fixed base edges and decrease linearly to zero at upper free edges of the walls. The obtained resulted values of wall deflections will be in terms of (isotropic rigidity (D), hydraulic density (ɤ) and walls height (h)).

The basic proposed model is based on transformed the fourth order differential equations to first order using FDM. Then solve these equations using the MATLAB R2018b package to obtain the elastic transverse deflection of walls. This model is better than the approach that used a very complicated Timoshenko and Woinowsky-Krieger based on theory of plates and shells.

Nomenclature
D flexural rigidity
q_z the applied liquid transvers load
υ the poison's ratio
E the modulus of elasticity
t the wall thickness
h the wall height
L the wall length.

2. Classical Plate Theory and Governing Equation

An analysis of a thin plate under the action to loads acting transvers to its surface requires solution of the differential equations of two-dimensional elasticity. The exact bending analysis for thin plates (Timoshenko and Woinowsky-Krieger, 1987) [1] is somewhat complex and time-consuming. To avoid mathematical difficulties associated with this exact equation, the Kirchhoff's classical theory of thin plates is used with sufficient accuracy in results without the need of carrying out a full two-dimensional stress analysis. The Kirchhoff's theory is expressed in terms of transverse deflections u(x, y) for which the governing differential equation is of fourth order, requiring only two boundary conditions to be satisfied at each edge. The governing diff. equs. for deflection of 2-D thin plates by Kirchhoff's theory can be presented as:

$$\nabla^4 u(x,y) = \frac{q_z(y)}{D} \quad (1)$$

Where,
x,y = Coordinate of node on the length of wall plate
u = Deflection in z direction transvers to walls plan
q_z = Applied triangular uniformly distributed liquid static load in z direction
∇^4 = Biharmonic differential operator= $\nabla^2 \nabla^2$
D = Flexural rigidity

Expanding the biharmonic operator the equation (1) can be simplified as [2]

$$\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2\,\partial y^2} + \frac{\partial^4 u}{\partial y^4} = \frac{q_z(y)}{D} \quad (2\text{-A})$$

The isotropic BM for walls are:

$$Mx = -D\left[\frac{\partial^2 u}{\partial x^2} + \upsilon\,\frac{\partial^2 u}{\partial y^2}\right] \quad (2\text{-B})$$

$$Mx = -D\left[\frac{\partial^2 u}{\partial y^2} + \upsilon\,\frac{\partial^2 u}{\partial x^2}\right] \quad (2\text{-C})$$

The isotropic SF for walls are:

$$Vx = -D\left[\frac{\partial^3 u}{\partial x^3} + (2-\upsilon)\frac{\partial^3 u}{\partial x\;\;\partial y^2}\right] \quad (2\text{-D})$$

$$Vy = -D\left[\frac{\partial^3 u}{\partial y^3} + (2-\upsilon)\frac{\partial^3 u}{\partial x^2\,\partial y}\right] \quad (2\text{-E})$$

The flexural rigidity can be defined by the expression [3]:

$$D = \frac{Et^3}{12(1-\upsilon^2)} \quad (3)$$

In this expression,

E = Modulus of elasticity of walls material
t = Thickness of wall plate
υ = Poisson's ratio for wall plate material [4-6]
The equation (2) is readily solvable using finite difference method.

3. The Central Finite Difference Method

The method of finite differences is considered one of the most important numerical techniques. The finite difference method (FDM) is a numerical method that solves the equations of boundary value problems using mathematical discretization. It is very viable numerical methods for solution of partial differential equation and hence is suitable for solving plated walls equation. This method is sufficiently accurate for thin plate analysis [7]. It is probably the most transparent and the most common method among the various numerical approaches available for analysis of thin plates. This technique eventually requires the solution of a system of linear algebraic equations.

The solution of the deflection in rectangular plated walls by using finite difference approach is achieved by dividing the plate into a square or rectangular mesh. By this way the problem is reduced to the simultaneous solution of a set of algebraic equations, written for every nodal point within the plate. To analyze a thin plate one needs to solve its established relations between loads and response deflections. The solution method was being numerical approach instead of the old analytical methods ones. On the other hand, in the numerical methods, the resulting equations including boundary conditions are solved in approximate numerical way. A numerical method is often preferred as it can be incorporated with computers easily. In this paper "Central Finite Difference Method", a convenient and straightforward numerical approach is used for the analysis of a thin plated walls. The innovative method can be easily programmed to readily apply on a plate issues.

3.1 FDM Expressions and Their Graphical Presentation

To apply the central finite difference method, the plated walls were divided in to eight by eight elements ΔX □ ΔY as can be seen from figure (1). Due to horizontal symmetry of walls with three fixed edges and free at top edge, one half of the walls were divided into eight by four elements with $\Delta X = L/8$ and $\Delta Y = h/8$. The plate divided into 32 real nodes and 8 some imaginary nodes on the left side, 4 under the bottom edge, and 16 to the right of the middle center line and 8 above the walls meshes are to be considered on the plate to be analyzed. The derivatives in the governing differential equations are then represented by difference quantities at the real nodes. The imaginary nodes on the left, bottom edges and that's located to the right of the vertical center line were replaced directly by its equivalent real nodes depend on the followed boundary conditions, but the other top imaginary nodes numbered from 33 to 40 will replaced by its equiva lent real nodes as will mention in boundary conditions article. The total number of equations will become only 40 equations instead of 68 (32 real and 36 imaginary) this because that all the imaginary nodes located on left, right and bottom replaced by its equivalent real nodes but that lies above top (numbers 33 to 40 will add to the first 32 real equations.). The nodes can be located at the joints of some imaginary square, rectangular reference network, called a finite difference mesh. Doing so, a package of algebraic equations are formed from which the plate deflections at the nodes can be deduced.

3.1.1 Node Generation

The nodes are generated by dividing the plate into some square meshes in case of h = L and rectangles meshes in other three cases. The length is divided into 8 equal parts L/8 and the width in 8 equal parts resulting in h/8 square or rectangular meshes as shown in figure (1). From structural symmetry point of view, there are some points with identical loading and boundary conditions. Taking it into consideration, total 40 numbers of unknown node deflections are found and they are designated as u1, u2,u3….. etc. At the nodes along the fixed edges deflections are marked as zero. To treat the boundary conditions some fictitious deflections are also shown outside the freely supported edge. Due to symmetrical conditions exist, only one half of the walls need to be considered. To solve the obtained linear equations for the wall deflection using preferably, an electronic computer.

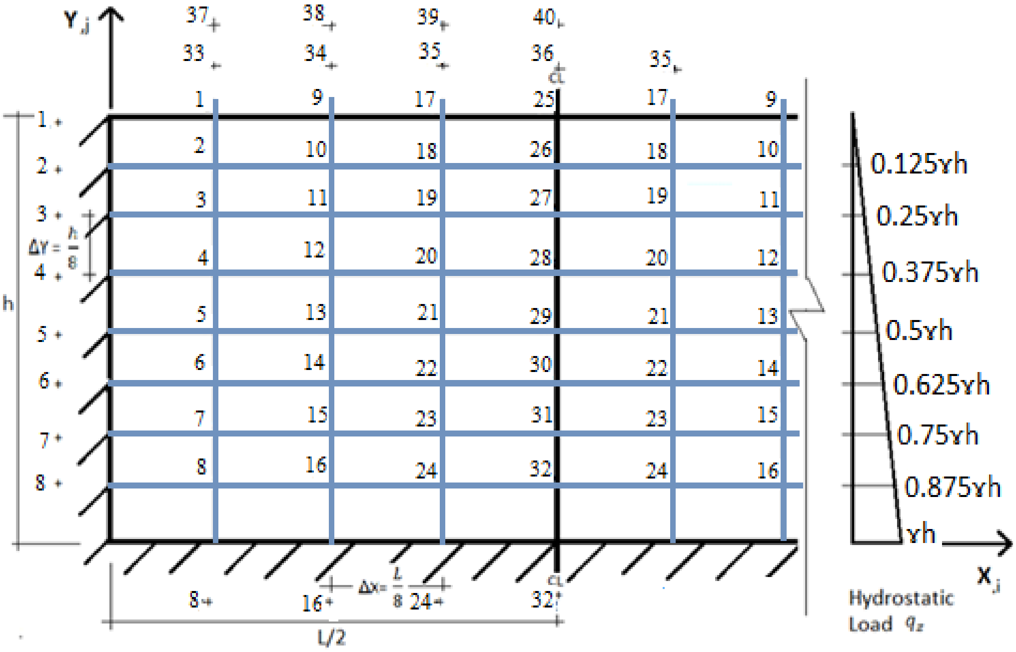


FIGURE 1 **Designation of Nodes Meshing and distributed Loads**

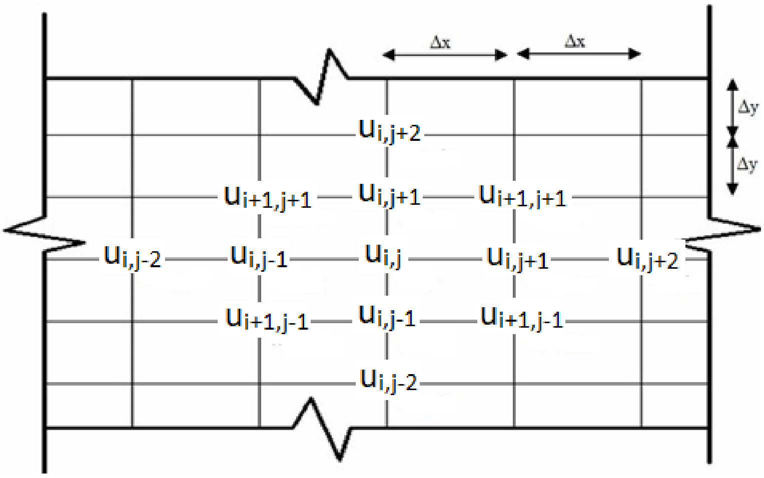


FIGURE 2 **Plate element with nodes and deflection components**

The FDM expressions required to solve the equation (2) at a node (i,j) in figure (2) are [2]:

$$u'_i = \frac{u_{i+1} - u_{i-1}}{2w}, \qquad u''_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{w^2}, \qquad u'''_i = \frac{u_{i+2} - 2u_{i+1} + 2u_{i-1} - u_{i-2}}{2w^3}$$

$$u^{iv} = \frac{u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}}{w^4}$$

Case 1:

L=h

$\Delta x = h/8 \Rightarrow (\Delta x)^4 = h^4/8^4$

$\Delta y = h/8 \Rightarrow (\Delta y)^4 = h^4/8^4$

$(\Delta x)^2(\Delta y)^2 = h^4/8^4$

Case 2:

L=2h

$\Delta x = 2h/8 \Rightarrow (\Delta x)^4 = 16\, h^4/8^4$

$\Delta y = h/8 \Rightarrow (\Delta y)^4 = h^4/8^4$

$(\Delta x)^2(\Delta y)^2 = 4h^4/8^4$

Case 3:

L=3h

$\Delta x = 3h/8 \Rightarrow (\Delta x)^4 = 81\ h^4/8^4$

$\Delta y = h/8 \Rightarrow (\Delta y)^4 = h^4/8^4$

$(\Delta x)^2(\Delta y)^2 = 9h^4/8^4$

Case 4:

L=4h

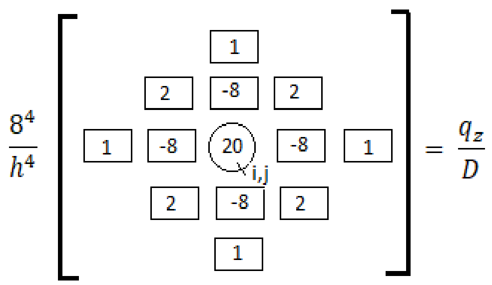
$\Delta x = 4h/8 \Rightarrow (\Delta x)^4 = 256\ h^4/8^4$

$\Delta y = h/8 \Rightarrow (\Delta y)^4 = h^4/8^4$

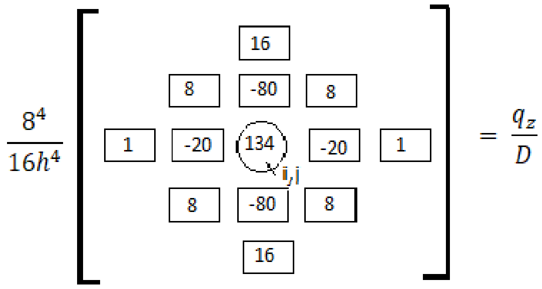
$(\Delta x)^2(\Delta y)^2 = 16h^4/8^4$

Here **Δx** is the spacing of nodes along the direction of (i) and **Δy** is the spacing of nodes along the direction of (**j**).

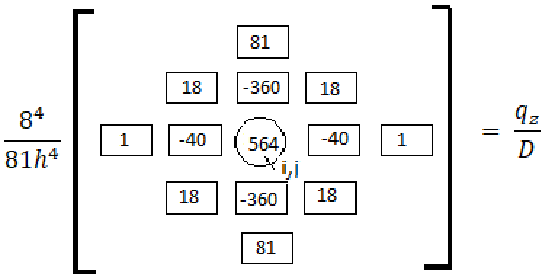
The derivative in equation (2), for the node (i,j) will replaced with the expression presented graphically in figure (2) [8], to get number of linear general equations (1 to 32) in matrix form.



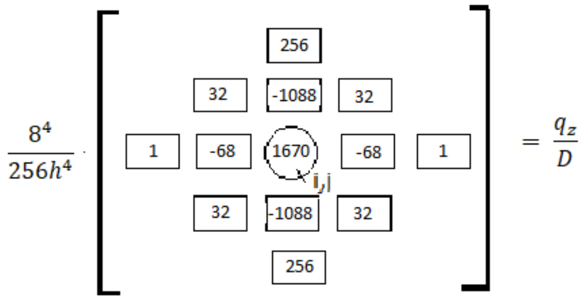
Case (1)



Case (2)



Case (3)



Case (4)

Case-1 $$Mx = \frac{-D\,8^2}{h^2}\begin{bmatrix} & 0.2 & \\ 1 & -2.4 & 1 \\ & 0.2 & \end{bmatrix} \qquad My = \frac{-D\,8^2}{h^2}\begin{bmatrix} & 1 & \\ 0.2 & -2.4 & 0.2 \\ & 1 & \end{bmatrix}$$

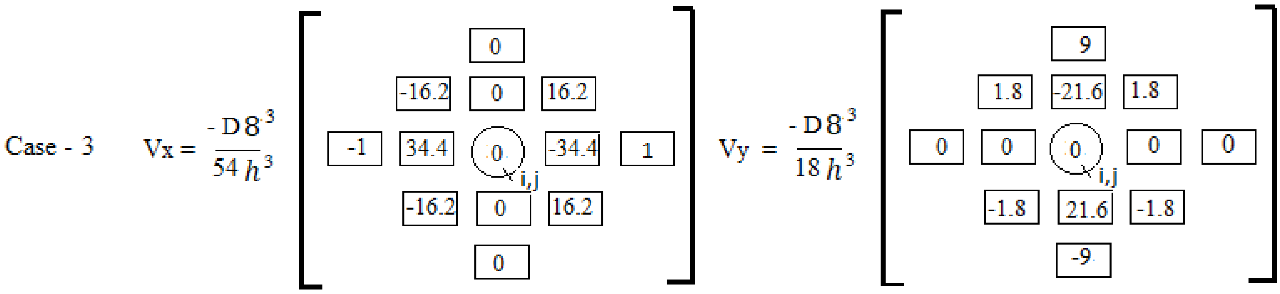
Case-2 $$Mx = \frac{-D\,8^2}{4\,h^2}\begin{bmatrix} & 0.8 & \\ 1 & -3.6 & 1 \\ & 0.8 & \end{bmatrix} \qquad My = \frac{-D\,8^2}{h^2}\begin{bmatrix} & 4 & \\ 0.2 & -8.4 & 0.2 \\ & 4 & \end{bmatrix}$$

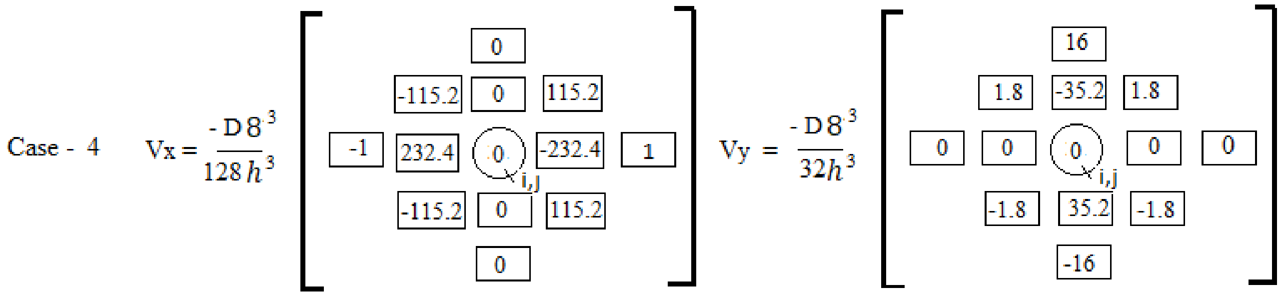
Case-3 $$Mx = \frac{-D\,8^2}{9\,h^2}\begin{bmatrix} & 1.8 & \\ 1 & -5.6 & 1 \\ & 1.8 & \end{bmatrix} \qquad My = \frac{-D\,8^2}{h^2}\begin{bmatrix} & 9 & \\ 0.2 & -18.4 & 0.2 \\ & 9 & \end{bmatrix}$$

Case-4 $$Mx = \frac{-D\,8^2}{16h^2}\begin{bmatrix} & 3.2 & \\ 1 & -8.4 & 1 \\ & 3.2 & \end{bmatrix} \qquad My = \frac{-D\,8^2}{h^2}\begin{bmatrix} & 16 & \\ 0.2 & -32.4 & 0.2 \\ & 16 & \end{bmatrix}$$

Case - 1 $$Vx = \frac{-D8^3}{2\,h^3}\begin{bmatrix} & & 0 & & \\ & -1.8 & 0 & 1.8 & \\ -1 & 5.6 & \underset{i,j}{0} & -5.6 & 1 \\ & -1.8 & 0 & 1.8 & \\ & & 0 & & \end{bmatrix} \qquad Vy = \frac{-D8^3}{2\,h^3}\begin{bmatrix} & & 1 & & \\ & 1.8 & -5.6 & 1.8 & \\ 0 & 0 & \underset{i,j}{0} & 0 & 0 \\ & -1.8 & 5.6 & -1.8 & \\ & & -1 & & \end{bmatrix}$$

Case - 2 $$Vx = \frac{-D8^3}{16\,h^3}\begin{bmatrix} & & 0 & & \\ & -7.2 & 0 & 7.2 & \\ -1 & 16.4 & \underset{i,j}{0} & -16.4 & 1 \\ & -7.2 & 0 & 7.2 & \\ & & 0 & & \end{bmatrix} \qquad Vy = \frac{-D8^3}{8\,h^3}\begin{bmatrix} & & 4 & & \\ & 1.8 & -11.6 & 1.8 & \\ 0 & 0 & \underset{i,j}{0} & 0 & 0 \\ & -1.8 & 11.6 & -1.8 & \\ & & -4 & & \end{bmatrix}$$





3.1.2 Treatment of Boundary Condition

To make use of equations or its corresponding graphical stencil in figure (2), the joint (i,j) must be accompanied with 12 surrounding joints. This possible for internal joints which have two joints before, two joints after, two joints above and two joints under the central joint (i,j), this was due to that the maximum derivatives degree are forth order in x and y directions. When apply the general equation in figure (2) on joint at or near edges some fictitious noted are imagined outside the plate boundary and proper boundary conditions are applied to express these fictitious deflections in terms of the deflection components inside the plate as below [9]:

The conditions to be applied for edges are:

1- Fixed end at left side (0,y)

a- For a point (i,j) on fixed edge: $u_{i,j} = 0$ and

b- $\frac{\partial u}{\partial x} = 0$ ⟶ $u_{i-1} = u_{i+1}$ (4)

Hence, the column of nodes numbers left to the left fixed edge (0,y) equals to that's to the right of it.

2- Fixed end at bottom (x,0)

a- For a point (i,j) on fixed edge: $u_{i,j} = 0$ and

b- $\frac{\partial u}{\partial y} = 0$ ⟶ $u_{j-1} = u_{j+1}$ (5)

Hence, the raw of nodes numbers under the bottom fixed edge (x,0) equals to that's to the upper of it.

3- At the centerline of width (L/2,y):

a- $\frac{\partial u}{\partial x} = 0$ ⟶ $u_{i+1} = u_{i-1}$ (6-A)

b- $\frac{\partial^3 u}{\partial x^3} = 0$ ⟶ $u_{i+2} = 2u_{i+1} - 2u_{i-1} + u_{i-2}$ (6-B)

Sub eq.(8) in eq.(B) gives:

$u_{i+2} = u_{i-2}$ (7)

Hence the numbers on imaginary nodes lying to the right of centerline were mirror to that to left of it.

4-1 Free end at top (x,h): The moment in vertical y-direction at the top free edge equals to zero and the domain for My will applied to nodes 1, 9, 17 and 25 for each of the four cases (eqs. Numbered 37 to 40 in the prepared matrix).

a- $My_{(x,h)} = 0$

Case -1:

for i = 1 $-2.4u_1 - u_2 + 0.2u_9 + u_{33} = 0$

for i = 9 $0.2u_1 - 2.4u_9 + u_{10} + 0.2u_{17} + u_{34} = 0$

for i = 17 $0.2u_9 - 2.4u_{17} + u_{18} + 0.2\,u_{25} + u_{35} = 0$

for i = 25 $0.4u_{17} - 2.4u_{25} + u_{26} + u_{36} = 0$

Case -2:

for i = 1 $-8.4u_1 - 4u_2 + 0.2u_9 + 4u_{33} = 0$

for i = 9 $0.2u_1 - 8.4u_9 + 4u_{10} + 0.2u_{17} + 4u_{34} = 0$

for i = 17 $0.2u_9 - 8.4u_{17} + 4u_{18} + 0.2u_{25} + 4u_{35} = 0$

for i = 25 $0.4u_{17} - 8.4u_{25} + 4u_{26} + 4u_{36} = 0$

Case -3:

for i = 1 $-18.4u_1 - 9u_2 + 0.2u_9 + 9u_{33} = 0$

for i = 9 $0.2u_1 - 18.4u_9 + 9u_{10} + 0.2u_{17} + 9u_{34} = 0$

for i = 17 $0.2u_9 - 18.4.4u_{17} + 9u_{18} + 0.2u_{25} + 9u_{35} = 0$

for i = 25 $0.4u_{17} - 18.4u_{25} + 9u_{26} + 9u_{36} = 0$

Case -4:

for i = 1 $-32.4u_1 - 16u_2 + 0.2u_9 + 16u_{33} = 0$

for i = 9 $0.2u_1 - 32.4u_9 + 16u_{10} + 0.2u_{17} + 16u_{34} = 0$

for i = 17 $0.2u_9 - 32.4u_{17} + 16u_{18} + 0.2u_{25} + 16u_{35} = 0$

for i = 25 $0.4u_{17} - 32.4u_{25} + 16u_{26} + 16u_{36} = 0$

4-2 Free end at top (x,h): The shear in vertical y-direction at the top free edge equals to zero and the domain for Vy will applied to nodes 1, 9, 17 and 25 for each of the four cases (eqs. Numbered 33 to 36 in the prepared matrix).

b- $Vy_{(x,h)} = 0$

Case -1:

for i = 1 $5.6u_2 - u_3 - 1.8u_{10} - 5.6u_{33} + 1.8u_{34} + u_{37} = 0$

for i = 9 $-1.8u_2 + 5.6u_{10} - u_{11} - 1.8u_{18} + 1.8u_{33} - 5.6.6u_{34} + 1.8u_{35} + u_{38} = 0$

for i = 17 $-1.8u_{10} + 5.6u_{18} - u_{19} - 1.8u_{26} + 1.8u_{34} - 5.6u_{35} + 1.8u_{36} + u_{39} = 0$

for i = 25 $-3.6u_{18} + 5.6u_{26} - u_{27} + 3.6u_{35} - 5.6u_{36} + u_{40} = 0$

Case -2:

for i = 1 $11.6u_2 - 4u_3 - 1.8u_{10} - 11.6u_{33} + 1.8u_{34} + 4u_{37} = 0$

for i = 9 $-1.8u_2 + 11.6u_{10} - 4u_{11} - 1.8u_{18} + 1.8u_{33} - 11.6u_{34} + 1.8u_{35} + 4u_{38} = 0$

for i = 17 $-1.8u_{10} + 11.6u_{18} - 4u_{19} - 1.8u_{26} + 1.8u_{34} - 11.6u_{35} + 1.8u_{36} + 4u_{39} = 0$

for i = 25 $-3.6u_{18} + 21.6u_{26} - 9u_{27} + 3.6u_{35} - 21.6u_{36} + 9u_{40} = 0$

Case -3:

for i = 1 $21.6u_2 - 9u_3 - 1.8u_{10} - 21.6u_{33} + 1.8u_{34} + 9u_{37} = 0$

for i = 9 $-1.8u_2 + 21.6u_{10} - 9u_{11} - 1.8u_{18} + 1.8u_{33} - 21.6u_{34} + 1.8u_{35} + 9u_{38} = 0$

for i = 17 $-1.8u_{10} + 21.6u_{18} - 9u_{19} - 1.8u_{26} + 1.8u_{34} - 21.6u_{35} + 1.8u_{36} + 9u_{39} = 0$

for i = 25 $-3.6u_{18} + 21.6u_{26} - 9u_{27} + 3.6u_{35} - 21.6u_{36} + 9u_{40} = 0$

Case -4:

for i = 1 $35.6u_2 - 16u_3 - 1.8u_{10} - 35.6u_{33} + 1.8u_{34} + 16u_{37} = 0$

for i = 9 $-1.8u_2 + 35.6u_{10} - 16u_{11} - 1.8u_{18} + 1.8u_{33} - 35.6u_{34} + 1.8u_{35} + 16u_{38} = 0$

for i = 17 $-1.8u_{10} + 35.6u_{18} - 16u_{19} - 1.8u_{26} + 1.8u_{34} - 35.6u_{35} + 1.8u_{36} + 16u_{39} = 0$

for i = 25 $-3.6u_{18} + 35.6u_{26} - 16u_{27} + 3.6u_{35} - 35.6u_{36} + 16u_{40} = 0$

4. Solution of FDM expressions by Matrix Method

By applying the FDM expression in equation (4) at all the nodes of the plate, a system of linear equations will be formed. Number of eqs. will be evidently equal to the num of point deflections and therefore, the system can be solved and deflections at all the nodes can be calculated out. Solution of the system by matrix method seems to be favorable as the system involves a large number of unknowns. To do so, a coefficient matrix needs to be formed and it is to be inversed. The inverse matrix is then multiplied to the right-hand side quantity in equation (4) to get the solution matrix. To create a matrix of equations so that each element was provided with correct finite difference equations LU numerical method was use in MATLAB package (2018) for solving the system of equations. The derivation of the operators is a simple matter to formulate the linear equations for deflection and solve them using LU method. To make it a practical method [10].

Case-1: L/h =1

$$9u_1 - 4u_2 + 2u_3 - 4u_9 + u_{17} = -\frac{0.5\gamma h^5}{D\beta^5}$$

$$-6u_1 + 20u_2 - 8u_3 + 1u_4 + 2u_9 - 8u_{10} + 2u_{11} + 1u_{18} = -\frac{1.5\gamma h^5}{D\beta^5}$$

$$1u_1 - 8u_2 + 21u_3 - 8u_4 + 1u_5 + 2u_{10} - 8u_{11} + 2u_{12} + u_{19} = -\frac{2.5\gamma h^5}{D\beta^5}$$

$$1u_2 - 8u_3 + 21u_4 - 8u_5 + 1u_6 + 2u_{11} - 8u_{12} + 2u_{13} + u_{20} = -\frac{3.5\gamma h^5}{DR^5}$$
$$1u_3 - 8u_4 + 21u_5 - 8u_6 + 1u_7 + 2u_{12} - 8u_{13} + 2u_{14} + u_{21} = -\frac{4.5\gamma h^5}{DR^5}$$
$$1u_4 - 8u_5 + 21u_6 - 8u_7 + 1u_8 + 2u_{13} - 8u_{14} + 2u_{15} + u_{22} = -\frac{5.5\gamma h^5}{DR^5}$$
$$1u_5 - 8u_6 + 21u_7 - 8u_8 + 2u_{14} - 8u_{15} + 2u_{16} + u_{23} = -\frac{6.5\gamma h^5}{DR^5}$$
$$1u_6 - 8u_7 + 22u_8 + 2u_{15} - 8u_{16} + u_{24} = -\frac{7.5\gamma h^5}{DR^5}$$

$$-4u_1 + 8u_9 - 4u_{10} + 2u_{11} - 4u_{17} + u_{25} = -\frac{0.5\gamma h^5}{DR^5}$$
$$2u_1 - 8u_2 + 2u_3 - 6u_9 + 19u_{10} - 8u_{11} + 1u_{12} + 2u_{17} - 8u_{18} + 2u_{19} + u_{26} = -\frac{1.5\gamma h^5}{DR^5}$$
$$2u_2 - 8u_3 + 2u_4 + 1u_9 - 8u_{10} + 20u_{11} - 8u_{12} + 1u_{13} + 2u_{18} - 8u_{19} + 2u_{20} + u_{27} = -\frac{2.5\gamma h^5}{DR^5}$$
$$2u_3 - 8u_4 + 2u_5 + 1u_{10} - 8u_{11} + 20u_{12} - 8u_{13} + 1u_{14} + 2u_{19} - 8u_{20} + 2u_{21} + u_{28} = -\frac{3.5\gamma h^5}{DR^5}$$
$$2u_4 - 8u_5 + 2u_6 + 1u_{11} - 8u_{12} + 20u_{13} - 8u_{14} + 1u_{15} + 2u_{20} - 8u_{21} + 2u_{22} + u_{29} = -\frac{4.5\gamma h^5}{DR^5}$$
$$2u_5 - 8u_6 + 2u_7 + 1u_{12} - 8u_{13} + 20u_{14} - 8u_{15} + 1u_{16} + 2u_{21} - 8u_{22} + 2u_{23} + u_{30} = -\frac{5.5\gamma h^5}{DR^5}$$
$$2u_6 - 8u_7 + 2u_8 + 1u_{13} - 8u_{14} + 20u_{15} - 8u_{16} + 2u_{22} - 8u_{23} + 2u_{24} + u_{31} = -\frac{6.5\gamma h^5}{DR^5}$$
$$2u_7 - 8u_8 + 1u_{14} - 8u_{15} + 21u_{16} + 2u_{23} - 8u_{24} + u_{32} = -\frac{7.5\gamma h^5}{DR^5}$$

$$u_1 - 4u_9 + 9u_{17} - 4u_{18} + 2u_{19} - 4u_{25} = -\frac{0.5\gamma h^5}{DR^5}$$
$$u_2 + 2u_9 - 8u_{10} + 2u_{11} - 6u_{17} + 20u_{18} - 8u_{19} + 1u_{20} + 2u_{25} - 8u_{26} + 2u_{27} = -\frac{1.5\gamma h^5}{DR^5}$$
$$u_3 + 2u_{10} - 8u_{11} + 2u_{12} + 1u_{17} - 8u_{18} + 21u_{19} - 8u_{20} + 1u_{21} + 2u_{26} - 8u_{27} + 2u_{28} = -\frac{2.5\gamma h^5}{DR^5}$$
$$u_4 + 2u_{11} - 8u_{12} + 2u_{13} + 1u_{18} - 8u_{19} + 21u_{20} - 8u_{21} + 1u_{22} + 2u_{27} - 8u_{28} + 2u_{29} = -\frac{3.5\gamma h^5}{DR^5}$$
$$u_5 + 2u_{12} - 8u_{13} + 2u_{14} + 1u_{19} - 8u_{20} + 21u_{21} - 8u_{22} + 1u_{23} + 2u_{28} - 8u_{29} + 2u_{30} = -\frac{4.5\gamma h^5}{DR^5}$$
$$u_6 + 2u_{13} - 8u_{14} + 2u_{15} + 1u_{20} - 8u_{21} + 21u_{22} - 8u_{23} + 1u_{24} + 2u_{29} - 8u_{30} + 2u_{31} = -\frac{5.5\gamma h^5}{DR^5}$$
$$u_7 + 2u_{14} - 8u_{15} + 2u_{16} + 1u_{21} - 8u_{22} + 21u_{23} - 8u_{24} + 2u_{30} - 8u_{31} + 2u_{32} = -\frac{6.5\gamma h^5}{DR^5}$$
$$u_8 + 2u_{15} - 8u_{16} + 1u_{22} - 8u_{23} + 22u_{24} + 2u_{31} - 8u_{32} = -\frac{7.5\gamma h^5}{DR^5}$$

$$2u_9 - 8u_{17} + 8u_{25} - 4u_{26} + 2u_{27} = -\frac{0.5\gamma h^5}{DR^5}$$
$$2u_{10} + 4u_{17} - 16u_{18} + 4u_{19} - 6u_{25} + 19u_{26} - 8u_{27} + 1u_{28} = -\frac{1.5\gamma h^5}{DR^5}$$
$$2u_{11} + 4u_{18} - 16u_{19} + 4u_{20} + 1u_{25} - 8u_{26} + 20u_{27} - 8u_{28} + 1u_{29} = -\frac{2.5\gamma h^5}{DR^5}$$
$$2u_{12} + 4u_{19} - 16u_{20} + 4u_{21} + 1u_{26} - 8u_{27} + 20u_{28} - 8u_{29} + 1u_{30} = -\frac{3.5\gamma h^5}{DR^5}$$
$$2u_{13} + 4u_{20} - 16u_{21} + 4u_{22} + 1u_{27} - 8u_{28} + 20u_{29} - 8u_{30} + 1u_{31} = -\frac{4.5\gamma h^5}{DR^5}$$
$$2u_{14} + 4u_{21} - 16u_{22} + 4u_{23} + 1u_{28} - 8u_{29} + 20u_{30} - 8u_{31} + 1u_{32} = -\frac{5.5\gamma h^5}{DR^5}$$
$$2u_{15} + 4u_{22} - 16u_{23} + 4u_{24} + 1u_{29} - 8u_{30} + 20u_{31} - 8u_{32} = -\frac{6.5\gamma h^5}{DR^5}$$
$$2u_{16} + 4u_{23} - 16u_{24} + u_{30} - 8u_{31} + 21u_{32} = -\frac{7.5\gamma h^5}{DR^5}$$

Case-1: L/h =2

$$39u_1 - 64u_2 + 32u_3 - 4u_9 + u_{17} = -\frac{8\gamma h^5}{D\beta^5}$$
$$-48u_1 + 119u_2 - 80u_3 + 16u_4 + 8u_9 - 20u_{10} + 8u_{11} + u_{18} = -\frac{24\gamma h^5}{D\beta^5}$$
$$16u_1 - 80u_2 + 135u_3 - 80u_4 + 16u_5 + 8u_{10} - 20u_{11} + 8u_{12} + u_{19} = -\frac{40\gamma h^5}{D\beta^5}$$
$$16u_2 - 80u_3 + 135u_4 - 80u_5 + 16u_6 + 8u_{11} - 20u_{12} + 8u_{13} + u_{20} = -\frac{56\gamma h^5}{D\beta^5}$$
$$16u_3 - 80u_4 + 135u_5 - 80u_6 + 16u_7 + 8u_{12} - 20u_{13} + 8u_{14} + u_{21} = -\frac{72\gamma h^5}{D\beta^5}$$
$$16u_4 - 80u_5 + 135u_6 - 80u_7 + 16u_8 + 8u_{13} - 20u_{14} + 8u_{15} + u_{22} = -\frac{88\gamma h^5}{D\beta^5}$$
$$16u_5 - 80u_6 + 135u_7 - 80u_8 + 8u_{14} - 20u_{15} + 8u_{16} + u_{23} = -\frac{104\gamma h^5}{D\beta^5}$$
$$16u_6 - 80u_7 + 151u_8 + 8u_{15} - 20u_{16} + u_{24} = -\frac{120\gamma h^5}{D\beta^5}$$

$$-4u_1 + 38u_9 - 64u_{10} + 32u_{11} - 4u_{17} + u_{25} = -\frac{8\gamma h^5}{D\beta^5}$$
$$8u_1 - 20u_2 + 8u_3 - 48u_9 + 118u_{10} - 80u_{11} + 16u_{12} + 8u_{17} - 20u_{18} + 8u_{19} + u_{26} = -\frac{24\gamma h^5}{D\beta^5}$$
$$8u_2 - 20u_3 + 8u_4 + 16u_9 - 80u_{10} + 134u_{11} - 80u_{12} + 16u_{13} + 8u_{18} - 20u_{19} + 8u_{20} + u_{27} = -\frac{40\gamma h^5}{D\beta^5}$$
$$8u_3 - 20u_4 + 8u_5 + 16u_{10} - 80u_{11} + 134u_{12} - 80u_{13} + 16u_{14} + 8u_{19} - 20u_{20} + 8u_{21} + u_{28} = -\frac{56\gamma h^5}{D\beta^5}$$
$$8u_4 - 20u_5 + 8u_6 + 16u_{11} - 80u_{12} + 134u_{13} - 80u_{14} + 16u_{15} + 8u_{20} - 20u_{21} + 8u_{22} + u_{29} = -\frac{72\gamma h^5}{D\beta^5}$$
$$8u_5 - 20u_6 + 8u_7 + 16u_{12} - 80u_{13} + 134u_{14} - 80u_{15} + 16u_{16} + 8u_{21} - 20u_{22} + 8u_{23} + u_{30} = -\frac{88\gamma h^5}{D\beta^5}$$
$$8u_6 - 20u_7 + 8u_8 + 16u_{13} - 80u_{14} + 134u_{15} - 80u_{16} + 8u_{22} - 20u_{23} + 8u_{24} + u_{31} = -\frac{104\gamma h^5}{D\beta^5}$$
$$8u_7 - 20u_8 + 16u_{14} - 80u_{15} + 150u_{16} + 8u_{23} - 20u_{24} + u_{32} = -\frac{120\gamma h^5}{D\beta^5}$$

$$u_1 - 4u_9 + 39u_{17} - 64u_{18} + 32u_{19} - 4u_{25} = -\frac{8\gamma h^5}{D\beta^5}$$
$$u_2 + 8u_9 - 20u_{10} + 8u_{11} - 48u_{17} + 119u_{18} - 80u_{19} + 16u_{20} + 8u_{25} - 20u_{26} + 8u_{27} = -\frac{24\gamma h^5}{D\beta^5}$$
$$u_3 + 8u_{10} - 20u_{11} + 8u_{12} + 16u_{17} - 80u_{18} + 135u_{19} - 80u_{20} + 16u_{21} + 8u_{26} - 20u_{27} + 8u_{28} = -\frac{40\gamma h^5}{D\beta^5}$$
$$u_4 + 8u_{11} - 20u_{12} + 8u_{13} + 16u_{18} - 80u_{19} + 135u_{20} - 80u_{21} + 16u_{22} + 8u_{27} - 20u_{28} + 8u_{29} = -\frac{56\gamma h^5}{D\beta^5}$$
$$u_5 + 8u_{12} - 20u_{13} + 8u_{14} + 16u_{19} - 80u_{20} + 135u_{21} - 80u_{22} + 16u_{23} + 8u_{28} - 20u_{29} + 8u_{30} = -\frac{72\gamma h^5}{D\beta^5}$$

$u_6 + 8u_{13} - 20u_{14} + 8u_{15} + 16u_{20} - 80u_{21} + 135u_{22} - 80u_{23} + 16u_{24} + 8u_{29} - 20u_{30} + 8u_{31} = -\frac{88\gamma h^5}{D8^5}$

$u_7 + 8u_{14} - 20u_{15} + 8u_{16} + 16u_{21} - 80u_{22} + 135u_{23} - 80u_{24} + 8u_{30} - 20u_{31} + 8u_{32} = -\frac{104\gamma h^5}{D8^5}$

$u_8 + 8u_{15} - 20u_{16} + 16u_{22} - 80u_{23} + 151u_{24} + 8u_{31} - 20u_{32} = -\frac{120\gamma h^5}{D8^5}$

$2u_9 - 8u_{17} + 38u_{25} - 64u_{26} + 32u_{27} = -\frac{8\gamma h^5}{D8^5}$

$2u_{10} + 16u_{17} - 40u_{18} + 16u_{19} - 48u_{25} + 118u_{26} - 80u_{27} + 16u_{28} = -\frac{24\gamma h^5}{D8^5}$

$2u_{11} + 16u_{18} - 40u_{19} + 16u_{20} + 16u_{25} - 80u_{26} + 134u_{27} - 80u_{28} + 16u_{29} = -\frac{40\gamma h^5}{D8^5}$

$2u_{12} + 16u_{19} - 40u_{20} + 16u_{21} + 16u_{26} - 80u_{27} + 134u_{28} - 80u_{29} + 16u_{30} = -\frac{56\gamma h^5}{D8^5}$

$2u_{13} + 16u_{20} - 40u_{21} + 16u_{22} + 16u_{27} - 80u_{28} + 134u_{29} - 80u_{30} + 16u_{31} = -\frac{72\gamma h^5}{D8^5}$

$2u_{14} + 16u_{21} - 40u_{22} + 16u_{23} + 16u_{28} - 80u_{29} + 134u_{30} - 80u_{31} + 16u_{32} = -\frac{88\gamma h^5}{D8^5}$

$2u_{15} + 16u_{22} - 40u_{23} + 16u_{24} + 16u_{29} - 80u_{30} + 134u_{31} - 80u_{32} = -\frac{104\gamma h^5}{D8^5}$

$2u_{16} + 16u_{23} - 40u_{24} + 16u_{30} - 80u_{31} + 150u_{32} = -\frac{120\gamma h^5}{D8^5}$

Case-1: L/h =3

$169u_1 - 324u_2 + 162u_3 - 4u_9 + u_{17} = -\frac{40.5\gamma h^5}{D8^5}$

$-198u_1 + 484u_2 - 360u_3 + 81u_4 + 18u_9 - 40u_{10} + 18u_{11} + u_{18} = -\frac{121.5\gamma h^5}{D8^5}$

$81u_1 - 360u_2 + 565u_3 - 360u_4 + 81u_5 + 18u_{10} - 40u_{11} + 18u_{12} + u_{19} = -\frac{202.5\gamma h^5}{D8^5}$

$81u_2 - 360u_3 + 565u_4 - 360u_5 + 81u_6 + 18u_{11} - 40u_{12} + 18u_{13} + u_{20} = -\frac{283.5\gamma h^5}{D8^5}$

$81u_3 - 360u_4 + 565u_5 - 360u_6 + 81u_7 + 18u_{12} - 40u_{13} + 18u_{14} + u_{21} = -\frac{364...5\gamma h^5}{D8^5}$

$81u_4 - 360u_5 + 565u_6 - 360u_7 + 81u_8 + 18u_{13} - 40u_{14} + 18u_{15} + u_{22} = -\frac{445.5\gamma h^5}{D8^5}$

$81u_5 - 360u_6 + 565u_7 - 360u_8 + 18u_{14} - 40u_{15} + 18u_{16} + u_{23} = -\frac{526.5\gamma h^5}{D8^5}$

$81u_6 - 360u_7 + 646u_8 + 18u_{15} - 40u_{16} + u_{24} = -\frac{607.5\gamma h^5}{D8^5}$

$-4u_1 + 168u_9 - 324u_{10} + 162u_{11} - 4u_{17} + u_{25} = -\frac{40.5\gamma h^5}{D8^5}$

$18u_1 - 40u_2 + 18u_3 - 198u_9 + 483u_{10} - 360u_{11} + 81u_{12} + 18u_{17} - 40u_{18} + 18u_{19} + u_{26} = -\frac{121.5\gamma h^5}{D8^5}$

$18u_2 - 40u_3 + 18u_4 + 81u_9 - 360u_{10} + 564u_{11} - 360u_{12} + 81u_{13} + 18u_{18} - 40u_{19} + 18u_{20} + u_{27} = -\frac{202.5\gamma h^5}{D8^5}$

$18u_3 - 40u_4 + 18u_5 + 81u_{10} - 360u_{11} + 564u_{12} - 360u_{13} + 81u_{14} + 18u_{19} - 40u_{20} + 18u_{21} + u_{28} = -\frac{283.5\gamma h^5}{D8^5}$

$18u_4 - 40u_5 + 18u_6 + 81u_{11} - 360u_{12} + 564u_{13} - 360u_{14} + 81u_{15} + 18u_{20} - 40u_{21} + 18u_{22} + u_{29} = -\frac{364...5\gamma h^5}{D8^5}$

$18u_5 - 40u_6 + 18u_7 + 81u_{12} - 360u_{13} + 564u_{14} - 360u_{15} + 81u_{16} + 18u_{21} - 40u_{22} + 18u_{23} + u_{30} = -\frac{445.5\gamma h^5}{D8^5}$

$18u_6 - 40u_7 + 18u_8 + 81u_{13} - 360u_{14} + 564u_{15} - 360u_{16} + 18u_{22} - 40u_{23} + 18u_{24} + u_{31} = -\frac{526.5\gamma h^5}{D8^5}$

$18u_7 - 40u_8 + 81u_{14} - 360u_{15} + 645u_{16} + 18u_{23} - 40u_{24} + u_{32} = -\frac{607.5\gamma h^5}{D8^5}$

$u_1 - 4u_9 + 169u_{17} - 324u_{18} + 162u_{19} - 4u_{25} = -\frac{40.5\gamma h^5}{D8^5}$

$u_2 + 18u_9 - 40u_{10} + 18u_{11} - 198u_{17} + 484u_{18} - 360u_{19} + 81u_{20} + 18u_{25} - 40u_{26} + 18u_{27} = -\frac{121.5\gamma h^5}{D8^5}$

$u_3 + 18u_{10} - 40u_{11} + 18u_{12} + 81u_{17} - 360u_{18} + 565u_{19} - 360u_{20} + 81u_{21} + 18u_{26} - 40u_{27} + 18u_{28} = -\frac{202.5\gamma h^5}{D8^5}$

$u_4 + 18u_{11} - 40u_{12} + 18u_{13} + 81u_{18}360u_{19} + 565u_{20} - 360u_{21} + 81u_{22} + 18u_{27} - 40u_{28} + 18u_{29} = -\frac{283.5\gamma h^5}{D8^5}$

$u_5 + 18u_{12} - 40u_{13} + 18u_{14} + 81u_{19} - 360u_{20} + 565u_{21} - 360u_{22} + 81u_{23} + 18u_{28} - 40u_{29} + 18u_{30} = -\frac{364...5\gamma h^5}{D8^5}$

$u_6 + 18u_{13} - 40u_{14} + 18u_{15} + 81u_{20} - 360u_{21} + 565u_{22} - 360u_{23} + 81u_{24} + 18u_{29} - 40u_{30} + 18u_{31} = -\frac{445.5\gamma h^5}{D8^5}$

$u_7 + 18u_{14} - 40u_{15} + 18u_{16} + 81u_{21} - 360u_{22} + 565u_{23} - 360u_{24} + 18u_{30} - 40u_{31} + 18u_{32} = -\frac{526.5\gamma h^5}{D8^5}$

$u_8 + 18u_{15} - 40u_{16} + 81u_{22} - 360u_{23} + 646u_{24} + 18u_{31} - 40u_{32} = -\frac{607.5\gamma h^5}{D8^5}$

$2u_9 - 8u_{17} + 168u_{25} - 324u_{26} + 162u_{27} = -\frac{40.5\gamma h^5}{D8^5}$

$2u_{10} + 36u_{17} - 80u_{18} + 36u_{19} - 198u_{25} + 483u_{26} - 360u_{27} + 81u_{28} = -\frac{121.5\gamma h^5}{D8^5}$

$2u_{11} + 36u_{18} - 80u_{19} + 36u_{20} + 81u_{25} - 360u_{26} + 564u_{27} - 360u_{28} + 81u_{29} = -\frac{202.5\gamma h^5}{D8^5}$

$2u_{12} + 36u_{19} - 80u_{20} + 36u_{21} + 81u_{26} - 360u_{27} + 564u_{28} - 360u_{29} + 81u_{30} = -\frac{283.5\gamma h^5}{D8^5}$

$2u_{13} + 36u_{20} - 80u_{21} + 36u_{22} + 81u_{27} - 360u_{28} + 564u_{29} - 360u_{30} + 81u_{31} = -\frac{364...5\gamma h^5}{D8^5}$

$2u_{14} + 36u_{21} - 80u_{22} + 36u_{23} + 81u_{28} - 360u_{29} + 564u_{30} - 360u_{31} + 81u_{32} = -\frac{445.5\gamma h^5}{D8^5}$

$2u_{15} + 36u_{22} - 80u_{23} + 36u_{24} + 81u_{29} - 360u_{30} + 564u_{31} - 360u_{32} = -\frac{526.5\gamma h^5}{D8^5}$

$2u_{16} + 36u_{23} - 80u_{24} + 81u_{30} - 360u_{31} + 645u_{32} = -\frac{607.5\gamma h^5}{D8^5}$

Case-1: L/h =4

$519u_1 - 1024u_2 + 512u_3 - 4u_9 + u_{17} = -\frac{128\gamma h^5}{D8^5}$

$-576u_1 + 1415u_2 - 1088u_3 + 256u_4 + 32u_9 - 68u_{10} + 32u_{11} + u_{18} = -\frac{384\gamma h^5}{D8^5}$

$$265u_1 - 1088u_2 + 1671u_3 - 1088u_4 + 265u_5 + 32u_{10} - 68u_{11} + 32u_{12} + u_{19} = -\frac{640\gamma h^5}{D8^5}$$
$$265u_2 - 1088u_3 + 1671u_4 - 1088u_5 + 265u_6 + 32u_{11} - 68u_{12} + 32u_{13} + u_{20} = -\frac{896\gamma h^5}{D8^5}$$
$$265u_3 - 1088u_4 + 1671u_5 - 1088u_6 + 265u_7 + 32u_{12} - 68u_{13} + 32u_{14} + u_{21} = -\frac{1152\gamma h^5}{D8^5}$$
$$265u_4 - 1088u_5 + 1671u_6 - 1088u_7 + 265u_8 + 32u_{13} - 68u_{14} + 32u_{15} + u_{22} = -\frac{1408\gamma h^5}{D8^5}$$
$$265u_5 - 1088u_6 + 1671u_7 - 1088u_8 + 32u_{14} - 68u_{15} + 32u_{16} + u_{23} = -\frac{1664\gamma h^5}{D8^5}$$
$$265u_6 - 1088u_7 + 1927u_8 + 32u_{15} - 68u_{16} + u_{24} = -\frac{1920\gamma h^5}{D8^5}$$

$$-4u_1 + 518u_9 - 1024u_{10} + 512u_{11} - 4u_{17} + u_{25} = -\frac{128\gamma h^5}{D8^5}$$

$$32u_1 - 68u_2 + 32u_3 - 576u_9 + 1414u_{10} - 1088u_{11} + 256u_{12} + 32u_{17} - 68u_{18} + 32u_{19} + u_{26} = -\frac{384\gamma h^5}{D8^5}$$

$$32u_2 - 68u_3 + 32u_4 + 256u_9 - 1088u_{10} + 1670u_{11} - 1088u_{12} + 256u_{13} + 32u_{18} - 68u_{19} + 32u_{20} + u_{27} = -\frac{640\gamma h^5}{D8^5}$$

$$32u_3 - 68u_4 + 32u_5 + 256u_{10} - 1088u_{11} + 1670u_{12} - 1088u_{13} + 256u_{14} + 32u_{19} - 68u_{20} + 32u_{21} + u_{28} = -\frac{896\gamma h^5}{D8^5}$$

$$32u_4 - 68u_5 + 32u_6 + 256u_{11} - 1088u_{12} + 1670u_{13} - 1088u_{14} + 256u_{15} + 32u_{20} - 68u_{21} + 32u_{22} + u_{29} = -\frac{1152\gamma h^5}{D8^5}$$

$$32u_5 - 68u_6 + 32u_7 + 256u_{12} - 1088u_{13} + 1670u_{14} - 1088u_{15} + 256u_{16} + 32u_{21} - 68u_{22} + 32u_{23} + u_{30} = -\frac{1408\gamma h^5}{D8^5}$$

$$32u_6 - 68u_7 + 32u_8 + 256u_{13} - 1088u_{14} + 1670u_{15} - 1088u_{16} + 32u_{22} - 68u_{23} + 32u_{24} + u_{31} = -\frac{1664\gamma h^5}{D8^5}$$
$$32u_7 - 68u_8 + 256u_{14} - 1088u_{15} + 1926u_{16} + 32u_{23} - 68u_{24} + u_{32} = -\frac{1920\gamma h^5}{D8^5}$$

$$u_1 - 4u_9 + 519u_{17} - 1024u_{18} + 512u_{19} - 4u_{25} = -\frac{128\gamma h^5}{D8^5}$$

$$u_2 + 32u_9 - 40u_{10} + 32u_{11} - 576u_{17} + 1415u_{18} - 1088u_{19} + 256u_{20} + 32u_{25} - 40u_{26} + 32u_{27} = -\frac{384\gamma h^5}{D8^5}$$

$$u_3 + 32u_{10} - 68u_{11} + 32u_{12} + 256u_{17} - 1088u_{18} + 1671u_{19} - 1088u_{20} + 256u_{21} + 32u_{26} - 68u_{27} + 32u_{28} = -\frac{640\gamma h^5}{D8^5}$$

$$u_4 + 32u_{11} - 68u_{12} + 32u_{13} + 256u_{18} - 1088u_{19} + 1671u_{20} - 1088u_{21} + 256u_{22} + 32u_{27} - 68u_{28} + 32u_{29} = -\frac{896\gamma h^5}{D8^5}$$

$$u_5 + 32u_{12} - 68u_{13} + 32u_{14} + 256u_{19} - 1088u_{20} + 1671u_{21} - 1088u_{22} + 256u_{23} + 32u_{28} - 68u_{29} + 32u_{30} = -\frac{1152\gamma h^5}{D8^5}$$

$$u_6 + 32u_{13} - 68u_{14} + 32u_{15} + 256u_{20} - 1088u_{21} + 1671u_{22} - 1088u_{23} + 256u_{24} + 32u_{29} - 68u_{30} + 32u_{31} = -\frac{1408 \gamma h^5}{D8^5}$$

$$u_7 + 32u_{14} - 68u_{15} + 32u_{16} + 256u_{21} - 1088u_{22} + 1671u_{23} - 1088u_{24} + 32u_{30} - 68u_{31} + 32u_{32} = -\frac{1664 \gamma h^5}{D8^5}$$

$$u_8 + 32u_{15} - 68u_{16} + 256u_{22} - 1088u_{23} + 1927u_{24} + 32u_{31} - 68u_{32} = -\frac{1920 \gamma h^5}{D8^5}$$

$$2u_9 - 8u_{17} + 518u_{25} - 1024u_{26} + 512u_{27} = -\frac{128 \gamma h^5}{D8^5}$$

$$2u_{10} + 64u_{17} - 136u_{18} + 64u_{19} - 576u_{25} + 1414u_{26} - 1088u_{27} + 256u_{28} = -\frac{384 \gamma h^5}{D8^5}$$

$$2u_{11} + 64u_{18} - 136u_{19} + 64u_{20} + 256u_{25} - 1088u_{26} + 1670u_{27} - 1088u_{28} + 256u_{29} = -\frac{640 \gamma h^5}{D8^5}$$

$$2u_{12} + 64u_{19} - 136u_{20} + 64u_{21} + 256u_{26} - 1088u_{27} + 1670u_{28} - 1088u_{29} + 256u_{30} = -\frac{896 \gamma h^5}{D8^5}$$

$$2u_{13} + 64u_{20} - 136u_{21} + 64u_{22} + 256u_{27} - 1088u_{28} + 1670u_{29} - 1088u_{30} + 256u_{31} = -\frac{1152 \gamma h^5}{D8^5}$$

$$2u_{14} + 64u_{21} - 136u_{22} + 64u_{23} + 256u_{28} - 1088u_{29} + 1670u_{30} - 1088u_{31} + 256u_{32} = -\frac{1408 \gamma h^5}{D8^5}$$

$$2u_{15} + 64u_{22} - 136u_{23} + 64u_{24} + 256u_{29} - 1088u_{30} + 1670u_{31} - 1088u_{32} = -\frac{1664 \gamma h^5}{D8^5}$$

5. Verifications of Proposed Model

The proposed model was verified through the comparisons with the exact solution presented by Timoshenko and Woinowsky-Krieger [1]. The case study chosen for that is a thin wall with h x h dimensions fixed in all edges and subjected to uniformly distributed load. MATLAB R2018 package is used for solving the proposed equations. The results showed that the maximum lateral deflection occurred at center of plate with value of 5.83q(h/8)4/D =0.0014 qh4/D according to Timoshenko and Woinowsky-Krieger and the maximum deflection was 0.00126 qh4/D according to the proposed equation using FDM. The ratio between two solutions was 0.90 which confirm the accuracy of the proposed model.

6. Parametric Study

The derived new equations are solved using MATLAB R2018b program to obtain the transverse deflection at nodes. The tank with fixed in three edges and have L=h, L=2h, L=3h, and L=4h were considered in this study.

Figures 3, 4, 5, and 6 show the 3-D model of transverse deflection in tank wall according to the ratio between the length and height of the wall. Figure 7, represents the maximum deflection behavior according to the ratio of L/h. Figure 8, shows the column chart for the maximum deflection for each L/h. Figure 9, shows the wall deflection in vertical centerline at the elevation for each L/h.

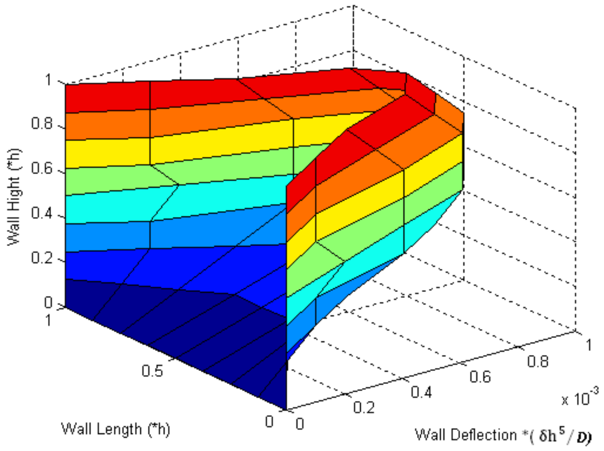


FIGURE 3 **Transvers Deflection for L=1h**

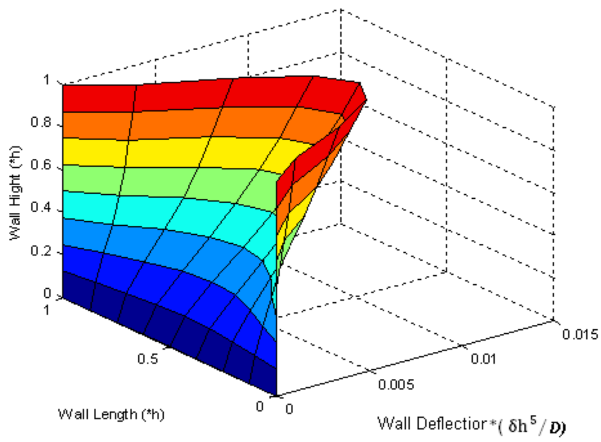


FIGURE 4 **Transvers Deflection for L=2h**

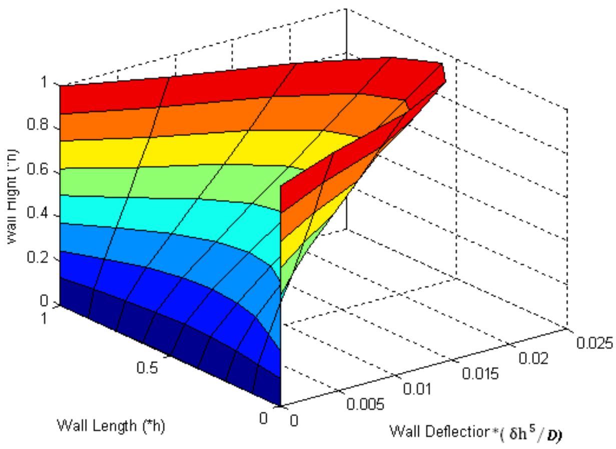


FIGURE 5 **Transvers Deflection for L=3h**

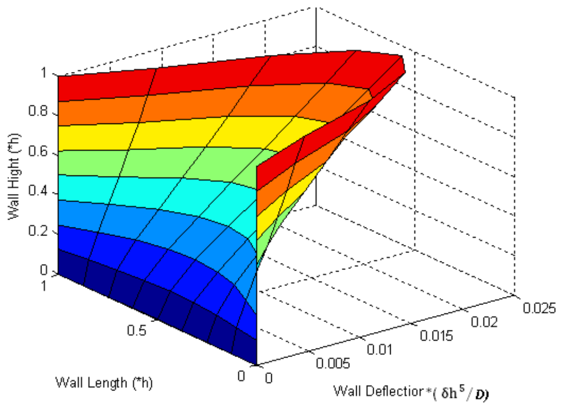


FIGURE 6 **Transvers Deflection for L=4h**

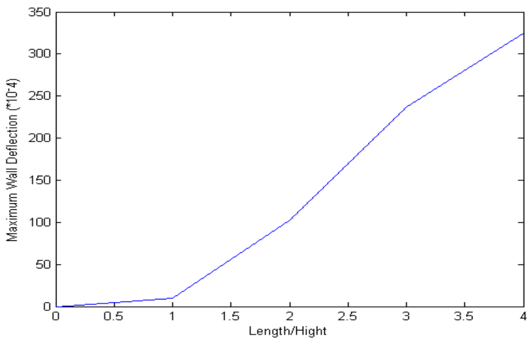


FIGURE 7 **Maximum Deflection for each L/h**

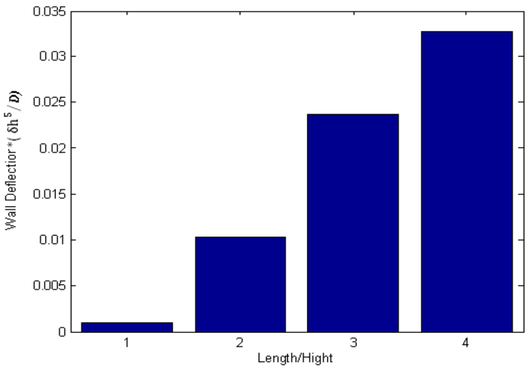


FIGURE 8 **Column Chart for Maximum Deflection For each L/h**

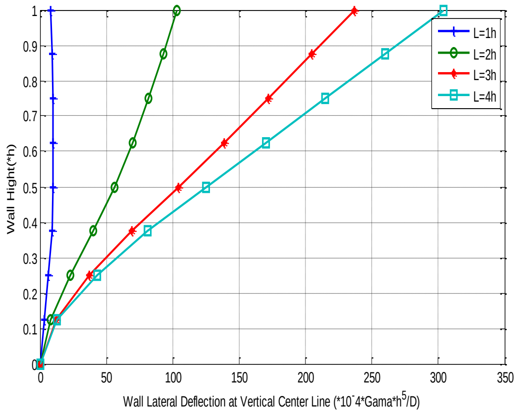


FIGURE 9 **Deflection of wall vertical centerline for each L/h**

7. Discussion of Results and Conclusions

1- The analysis indicates that the maximum deflection for tank wall occurred at the center of free edge (L/2,h) for L=2h, 3h and 4h but for case L=h the maximum deflection was near the mid height of wall (L/2,h/2).

2- The percentage of increasing in maximum deflection in tank wall was 10.3% and 23.7% and 32.5% in L/h=2, 3 and 4 compares with L/h=1.

3- As expected, the maximum deflection increased with increasing the ratio of L/h [11].

4- Through the results extracted from the analysis, it can be concluded that walls with square or semi-square shapes are the best, safest and least exposed to the problems of heavy precipitation and the resulting cracks in the granite [12].

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CONFLICTS OF INTEREST

The authors declare no conflict of interest

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