

INVESTIGATING OF THE CRITICAL THERMAL BUCKLING LOAD OF LINEAR FUNCTIONALLY GRADED RECTANGULAR FIN

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ABSTRACT: This study investigates the critical buckling due to heat Load of rectangular functionally graded material fins (FG-Fins) under uniform temperature conditions. This study differs from traditional studies by looking at graded properties in the axial direction. In this case, the mechanical and thermal properties (modulus of elasticity, Poisson ratio and thermal expansion coefficient) were graded in the length direction of the fins, which in this study the upper surface of the fins were considered to be pure copper. A finite element model in three dimensions was created in ANSYS APDL with the 10-node tetrahedral element (SOLID87) using a stepped-property approach. Fixed–Fixed and Simply Supported–Fixed boundary conditions were exposed to stepwise static thermal analysis followed by eigenvalue buckling analysis. The model which was validated against published studies showed a high agreement with the error of less than 5.654%. The critical thermal buckling Load was seen to rise with reduced length-to-width ratios and increased elasticity modulus ratios. Under axial gradation of mechanical properties, the effect of thermal stability was more significant than the effect of geometric aspect ratios. Fixed–Fixed condition showed the highest buckling resistance comparing with Simply Supported–Fixed condition. While the variation of length-to-width ratios were paramount in determining the form of higher modes.

Keywords: Functionally Graded Material, Linear Model, Thermal Buckling Load, Temperature Distribution, FG-

Fin



NOMENCLATURE	
a. English	
AL ₂ O ₃	Alumina
APDL	ANSYS Parametric Design Language
Cu	Copper
DL	Length of an individual discretized part/segment of the FG-Fin
E	(Young's modulus)
E _{Ratio}	Modulus of elasticity Ratio
FGM	Functionally Graded Material
FG-Fin	Functionally Graded Fin
L	Length of the FG-Fin / FG-plate
L/W	Length-to-width ratio (aspect ratio)
Ni	Nickel
P	Material property at any given point y along the length
P(0)	Material property at the bottom/base of the fin
P(L)	Material property at the top surface/tip of the fin

SOLID87	10-node 3D tetrahedral element in ANSYS APDL
t	Thickness of the FG-Fin / FG-plate
W	The breadth of the FG-Fin / FG-plate
y	Align along the fin's length (axial direction).
y/L	Dimensionless length
1D/2D/3D	One-, Two-, and Three-Dimensional
b. Latin	
α	Coefficient of thermal expansion
β	Slope/factor of the linear property variation equation
ν	Poisson's ratio

1. INTRODUCTION

The first appearing of functionally graded material was in Japan 1984, to achieve the idea of overcoming the jump in material properties and thermal stress in the traditional composite materials by varying its properties smoothly. The Functionally graded material is defined as "the materials are an advanced category of heterogeneous composite materials with multifunctional characteristics by gradually changing composition in one or more directions"[1,2] or "non-homogeneous composite materials where the composition smoothly and continuously changes in a certain direction, as along the thickness or length" [3,4]. The material properties are graded along 1D (such as thickness), 2D (such as length and width) or 3D (length, width and thickness) for improving lightweight, high strength, corrosion resistance, toughness and / or wear resistance [1] [5].

Because of smooth transition in material properties through the dimensions (1D, 2D or 3D) of FGM, the delamination failure (that appears in conventional laminated composites) is overcoming [1] [5]. Therefore, FGMs are used in different engineering applications such as energy absorption systems [1] transducers/sensors [6], optical electronics, and nuclear power generation.

In 1D-FGM, the distribution scheme of material properties can divided into two types: (a) Thickness or Height FGM: The fluctuation of material characteristics along the cross section area of plate or beam (b) Axial FGM: the change in material characteristics throughout a plate or beam's length. According to the distribution scheme Properties of the material, the thermal - structural actions of 1D-FGMs depends on this distribution scheme. Therefore, two cases in thermal - structural behavior must studied; the first one is temperature distribution case considering heat transfers problem in FGM plate or beam and the displacement distribution along the plate or beam considering the type of structural problem (static, buckling or vibration).

Sobamowo et al. [7] utilized finite difference technique to analyze and investigate the thermal performance of a convective-radiative FG-fin using four models (linear, quadratic, exponential, and power-law models) to describe the distribution of material properties. In other side, The transient thermal nonlinear problem of a convective-radiative FG-fin under a magnetic field was studied by Sobamowo et al. [8] and they found that the performance of FG-fin improved enhancing the "inhomogeneity- index" of radiative and magnetic field factors.

George et al. [9] used power law model for description the distribution of material properties of FGM in order to Examine the influence of convective and radiative factor on the heat sink functionality of porous FG-Fin using "differential transform method". In other side, Oguntala et al. [10] used exponential model to describe. The change in material characteristics along the longitudinal direction for calculating temperature distribution of the FG-fin. They created a novel methodology by applying Laplace transform and Chebyshev Pseudospectral collocation technique to investigate the transient thermal performance of a convective-radiative FG-fin subjected to "Lorenz force effects".

By combining Laplace transformation and collocation of Legendre wavelets techniques, Jemiseye et al. [11] created a hybrid computational for studying space and time based not linear heat models of a longitudinally conductive-radiative FG extended surface assuming quadratic, linear, and exponential heat conductivity models. Also, Jemiseye et al. [12] used numerical method and analytical method to analyze. The issue with heat transfer of rectangular FG-fin. Adnan A. Hussein et al. [13] investigated the effect of linear model on the temperature profile of axial FG-fin applying analytical method and a Finite-Element method.

Due to the limitations of manufacturing methods, the FG-beams and FG-plates are usually made as a layered assuming step-function to define the material properties of layers. The distribution's function and layer's number are important factors affected on designing and manufacturing FG-beams and FG-plates to decrease stress concentrations created due to vary material properties and geometric discontinuities, respectively [14]. Several studies dealing with buckling problem of FG- plate were made such as:

A.M. Zenkour and M. Sobhy [15] studied the thermal buckling of simply supported sandwich FG-plates (the core is homogeneous and isotropic and A power-law model is used to grade the face sheets according to thickness). They assumed that " transverse shear effects were captured applying a theory of sinusoidal shear deformation ". They found that "while thicker cores improved thermal stability, the critical buckling temperature dropped with increased aspect ratio and lowering thickness". Mostapha et.al [16] used "higher-order shear deformation theory" to investigate the

thermal buckling problem of simply supported rectangular FG-plates. They used power-law model to grade the material properties across the thickness. They also used ANSYS software to validate the projected critical buckling temperatures, and the results showed good agreement. They found that "while increasing the aspect ratio improved thermal buckling resistance, FGPs showed lower critical temperatures than homogenous plates".

L.W. et.al [17] used a temperature-dependent power-law model to grade the characteristics of the material across thickness in order to investigate the thermal buckling that is nonlinear and flutter behaviors of a FG-panel under aerodynamic and thermal loads. They used "First-order piston theory", "von Kármán geometrical nonlinearity" and "First shear deformation theory" to model this problem and also used the "Newton-Raphson iterative approach" to solve the "nonlinear finite element equations" under different boundary conditions. Yufeng et.al [18] proposed the critical buckling temperature using a novel approach of rectangular thin FG-plate under simply supported, clamped, and directed boundary conditions. They used separation-of-variables method to develop closed-form analytical solutions to calculate the critical buckling temperature and they described the variation of material properties along the thickness as a power-law model. They showed that "the critical temperature increased with the length-to-width ratio and reduced with the length-to-thickness ratio and power-law index. The uniform temperature field had the biggest impact on thermal stability". Dragan et.al [19] using a power-law model to describe the material properties along the thickness of thick moderately thick rectangular FG-plates to calculate the critical buckling temperature by applying high-order shear deformation theory and von Kármán nonlinear formulation.

Y.H. Dong and Y.H. Li [20] used several distribution models to describe the gradation along the thickness of FGM for investigating the thermal buckling behavior of rectangular FG-plates with simply supported. They based on higher-order shear deformation theory to derive the governing equations for bending, buckling, and vibration considering the effect of geometric nonlinearity and mid-plane stretching. They found that that thermal loads and material heterogeneity significantly affected structural responses, with thicker plates exhibiting higher buckling loads and vibration frequencies decreasing as thermal load increased. Vuong et.al [21] used a refined quasi-3D higher-order shear deformation theory and Meshless radial point interpolation approach to study the thermal buckling problem of FG-plates method considering the effect of transverse shear and thickness stretching. They calculated critical buckling temperatures and modes of FG- plates with different aspect ratios, length-to-thickness ratios, and boundary conditions under non-uniform and surface heat loads. They found that "the suggested approach effectively captured thermal buckling over thin to thick plates, and ignoring thickness stretching overestimates critical temperatures". Vuong et.al [22] used higher-order shear deformation theory, von Kármán nonlinearity and two transverse shear functions to study the nonlinear thermal buckling of FG- plates assuming the material properties are graded along the thickness. They applied the "modified Newton-Raphson scheme" to solve the nonlinear equations, and compared the numerical model with analytical solutions.

M. Mohammadi et.al [23] studied the buckling, static, and dynamic behavior of incompressible simply supported thick FG-plates assuming the material properties varied along the thickness according to power-law model. They used higher-order shear and normal deformable plate theory, Legendre polynomial expansion and variational approach, to derive governing equations for static, stability and dynamic analysis of FG-plate. Thamires X et.al [24] investigated the thermal buckling and post-buckling behavior of FG- plates whose material properties vary continuously with thickness according to power law model. They applied the finite element method by using Abaqus software to calculate critical buckling temperatures and buckling modes of FG- plates with different boundary conditions and geometric ratios. They found that "while increasing ceramic volume fractions improved thermal stability and decreased displacements, thinner plates buckled with lower temperature changes".

Farrokhi et. al [25] studied the heat buckling of rectangular FG-plates along the thickness direction using finite element method framework and applying "Carrera unified formulation and higher-order plate theory". They found the optimum critical buckling temperature of FG- plate according to geometric ratios and the boundary conditions of FG-plate. Majid Afzali et.al [26] investigated linear thermal buckling of rectangular FG-plates with dependent on temperature characteristics using the Carrera Unified Formulation (CUF) and an enhanced finite element model to capture accurate temperature distribution

From a previous review of thermal buckling studies, the material properties of FG-plate change across the thickness direction, using power law model and numerical methods. In thermal analysis studies, the temperature distribution of axial-FG is calculated using analytical method by assuming linear model to represent the material properties variation along the longitudinal direction. The power law, exponential, and/or sigmoid models cannot apply in the analytical solution of thermal studies because of the difficulty of solving governing equations. Finally, the use of thermal analysis to estimate the critical thermal buckling load of an axially FG-fin assuming linear model is the main objective of this work. In this work, the Material attributes are graded. along the length of FG-Fin assuming linear model. This FG-Fin is supported from top and bottom edges as fixed-fixed and simply supported-fixed FG-Fin. The effect of modulus ratio, thermal expansion ratio and length-to-width ratio on the critical thermal buckling load are studied in this work using ANSYS software.

2. PROBLEM DESCRIPTION

The FG-Fin is used in several engineering application in order to enhance the heat transfer problem and reduce the thermal stresses and avoid the thermal buckling behavior. The FG-fin has dimensions ($L \times w \times t$) as shown in Figure 1. The FG-fin (i.e. FG-plate) is supported using two types of supported (Fixed-Fixed case and Simply supported-Fixed case). In both cases, the right and left edges are free and the bottom edge of FG-Fin is fixed. While the top edge is fixed for the fixed-fixed case and simply supported for the simply supported-fixed case.

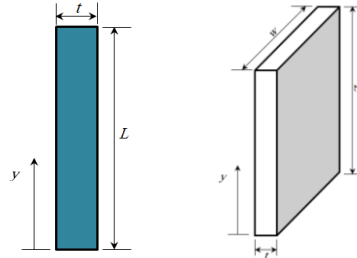


FIGURE 1 Geometry of FG-Fin

3. MATERIAL PROPERTIES MODEL

Several models are used to describe the material properties of FGM according to graduation direction such as exponential model, sigmoid model and power law model [3,4]. In this work, the length direction (i.e. axial direction) is the graduation direction. The material properties (such as thermal expansion coefficient, modulus of elasticity) of FG-Fin are described using the "Linear model" as shown in Eq. (1):

$$p(y) = p_o(1 + \beta y) \quad (1)$$

Where $P(y)$: The property at any point (y) along the length of plate, $P_0 = P(0)$; The property at point ($y=0$), $P(L)$;

The property at point ($y=L$); β : The slop of linear equation ($\beta = \frac{P(0)}{P(L)} - 1$).

4. ANSYS MODEL OF THERMAL BUCKLING PROBLEM

In this model, the graduated material properties is changed to stepped material properties. In other word, the non-homogenous FG-fin (one part) is represented as number of homogenous material parts. In this work, the FG-fin is separated into twenty parts (dividing the length of FG-Fin into (10) parts and dividing the width of FG-Fin into (2) parts). According to dividing the length of FG-Fin into (10) parts, (10) sets of material properties must calculated. The following steps describe the present ANSYS model:

1-Starting ANSYS software and Selecting the Suitable Element: Figure (2) shows the steps used to start the ANSYS APDL software (version 17.2) and select the suitable element (the tetrahedral element with 10 nodes (SOLID87) is used in this work).

2-Material Properties: The following formula is used to calculate material properties of every part of the FG-fin (in this study the FG-fin is divided into (20) parts, the length is divided into (10) parts and width is divided into (2) parts) assuming each part is a homogeneous material. Each material properties of each part along the length are calculated as follows: $P_i = (P(i \cdot DL) + P((i-1) \cdot DL)) / 2 \quad i=1, 2, \dots, 10$ (2)

where (DL) is the length of each portion of FG-Fin and " P_i " is the property of the i -th part of FG-Fin. Ten sets of material attributes are inserted into ANSYS APDL software by this model using the procedures outlined in Figure (3).

3-Drawing the FG-fin: The FG-fin is split into (20) parts (the FG-fin is divided into (20) parts, the length is divided into (10) parts and width is divided into two parts), each part of FG-fin has the following dimensions: ($L/10$) length, ($w/2$) width, and (t) thickness and the FGM-plate is drawn using steps illustrated in Figure (4).

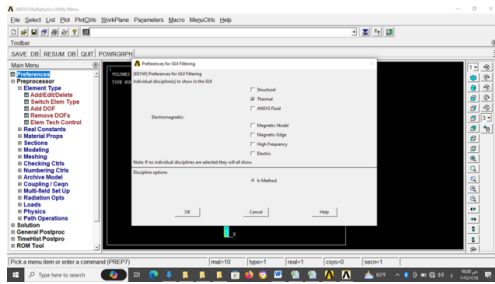
4-The created volumes (number of volumes is 20) are meshed using the selected element as shown in Figure (5).

5-In the Fixed – Fixed case, two boundary conditions are used. The first one represents the fixed support at the bottom edge. While the second one represents the fixed support at the top edge as shown in Figure (6-a and b). In the simply supported-Fixed case, the first one represents the fixed support at the bottom edge as shown in Figure (6-c and d), while the second one represents the simply support at the upper edge as shown in Figure (6-e and j).

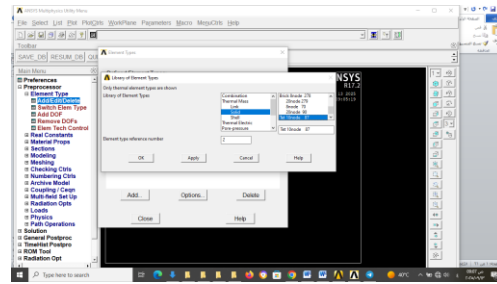
6- In thermal buckling problem, the thermal load is applied using the step illustrated in Figure (7)

7-The solving of this model require two types of solutions. The first one is static solution (see Figure (8-a to d)). While the second solution is Eigen Buckling solution which gives the critical buckling of heat load s and the mode of deformation (see Figure (8-e to h)).

8-In order to display the critical thermal buckling loads (temperature), the steps illustrated in Figure (9) are followed.

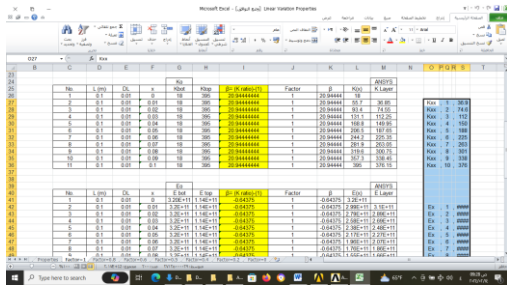


(a) Start.

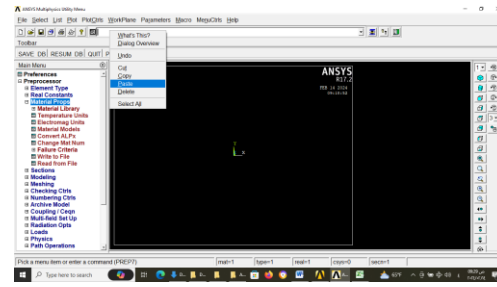


(b) Select Element.

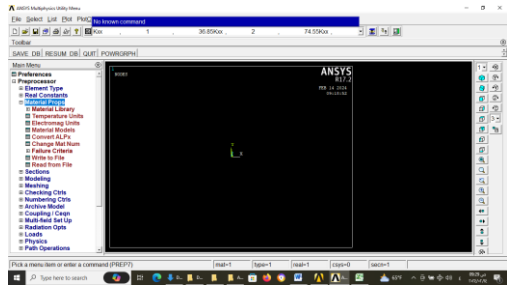
FIGURE 2 Starting ANSYS Software and Selecting the suitable Element



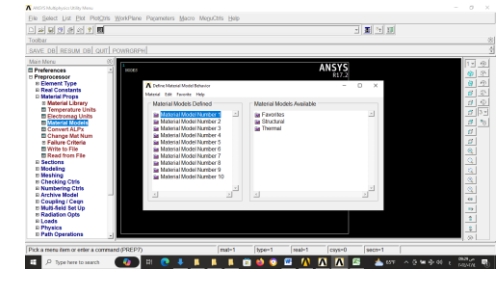
(a) Input Material Properties Sets-step1.



(b) Input Material Properties Sets-step2.

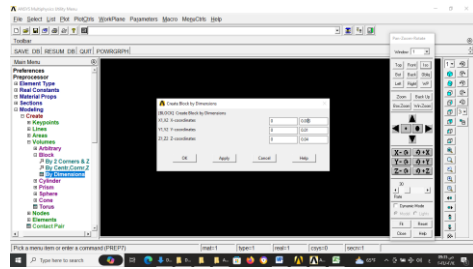


(c) Input Material Properties Sets-step3.

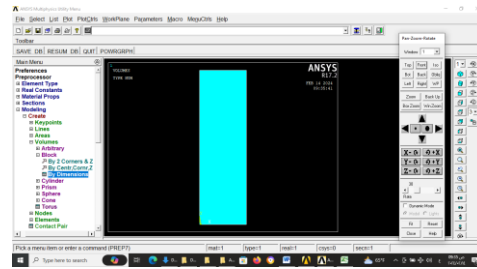


(d) Input Material Properties Sets-step4.

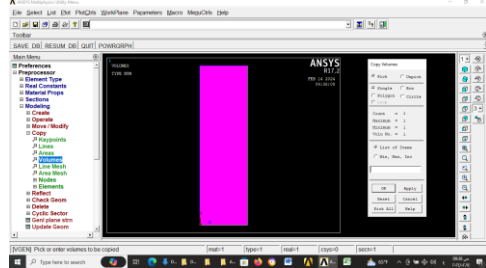
FIGURE 3 Steps of Input the Material Properties Sets of FG-Fin



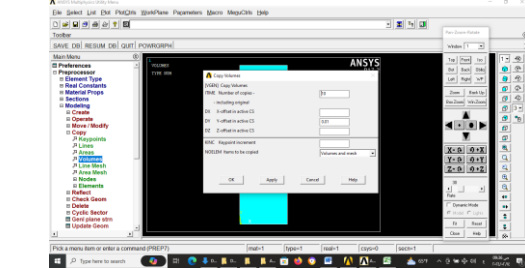
(a) Drawing the Geometry- Step1.



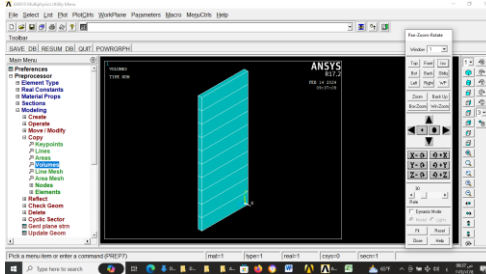
(b) Drawing the Geometry- Step2.



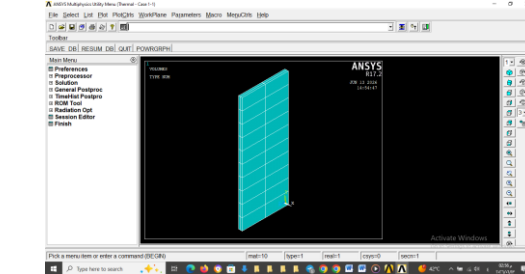
(c) Drawing the Geometry- Step3.



(d) Drawing the Geometry- Step4.

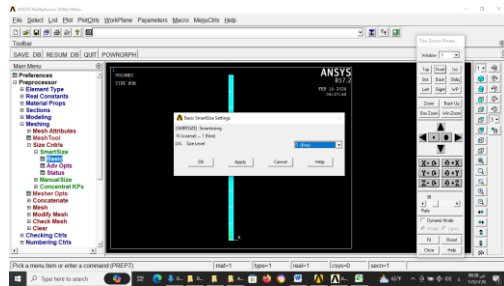


(e) Drawing the Geometry- Step5.

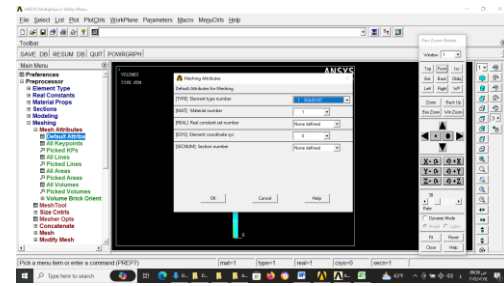


(f) Drawing the Geometry- Step6.

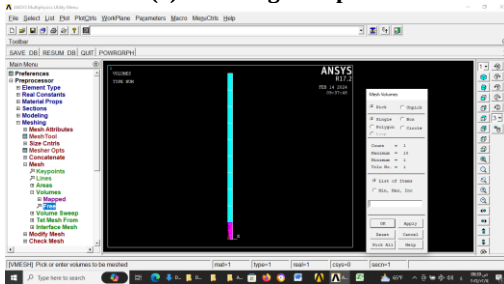
FIGURE 4 Steps of Drawing FG Fin



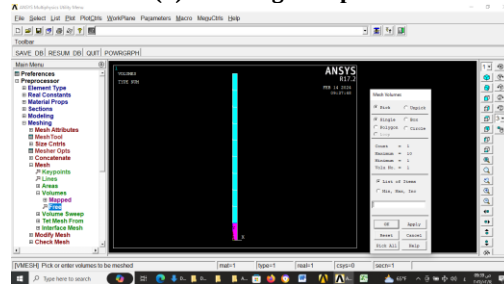
(a) Meshing – Step1.



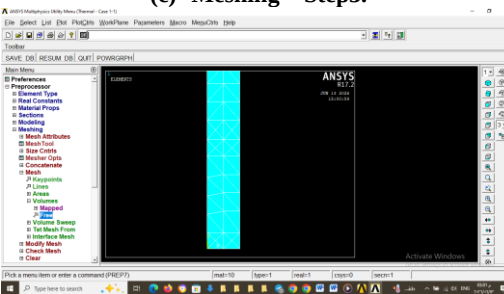
(b) Meshing – Step2.



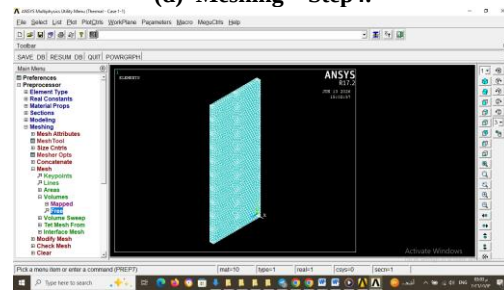
(c) Meshing – Step3.



(d) Meshing – Step4.

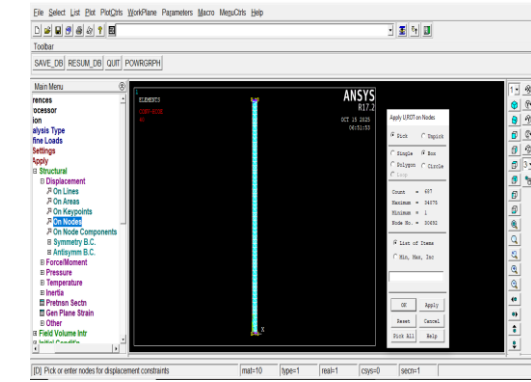


(e) Meshing – Step5.

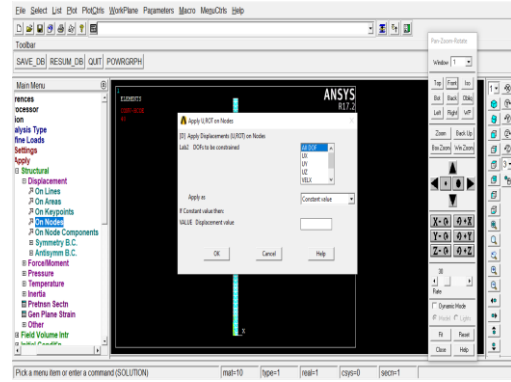


(f) Meshing – Step6.

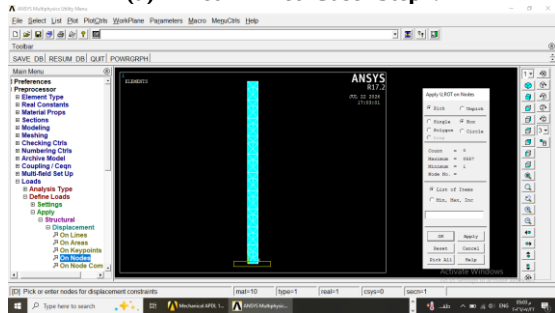
FIGURE 5 Steps of Meshing FG Fin



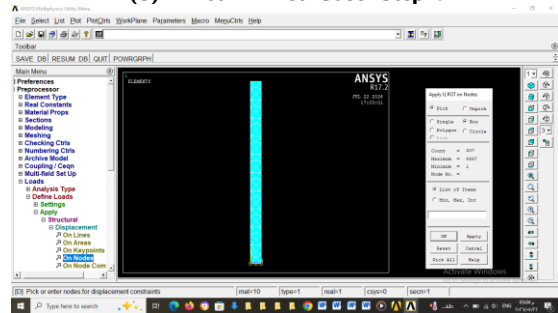
(a) Fixed – Fixed Case -Step1.



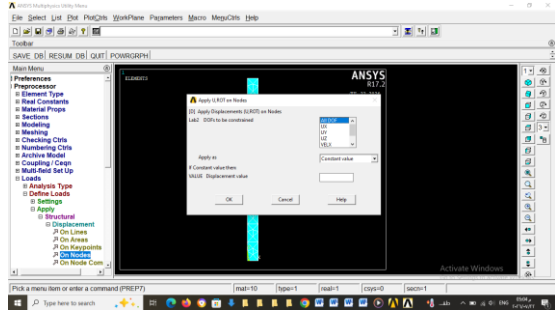
(b) Fixed – Fixed Case -Step2.



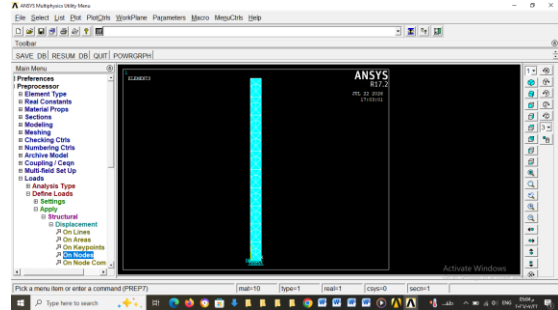
(c) Simply Supported – Fixed Case-Step3.



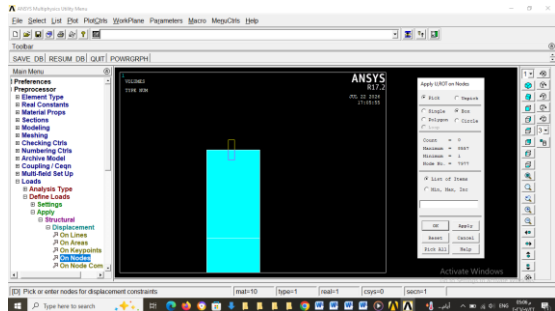
(d) Simply Supported – Fixed Case-Step4.



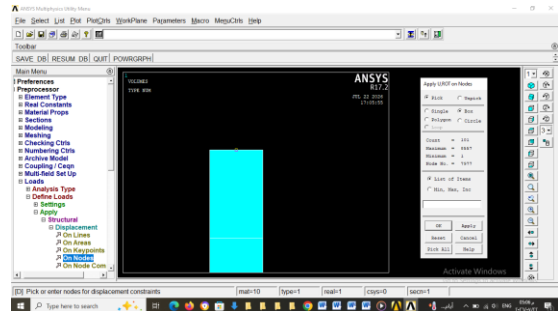
(e) Simply Supported – Fixed Case-Step5.



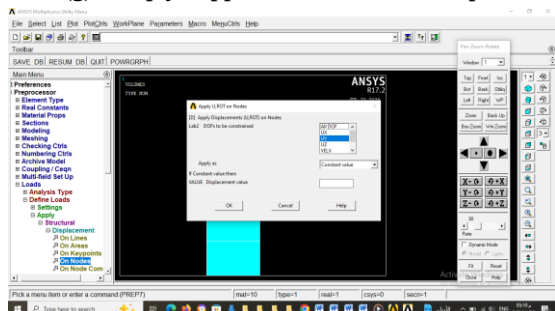
(f) Simply Supported – Fixed Case-Step6.



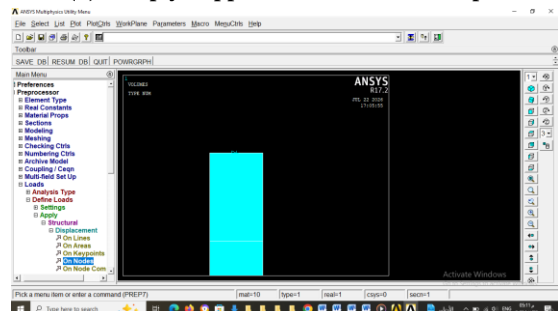
(g) Simply Supported – Fixed Case-Step7.



(h) Simply Supported – Fixed Case-Step8.



(i) Simply Supported – Fixed Case-Step9.



(j) Simply Supported – Fixed Case-Step10.

FIGURE 6 Steps of Applying Structural Boundary Conditions of the Two Cases

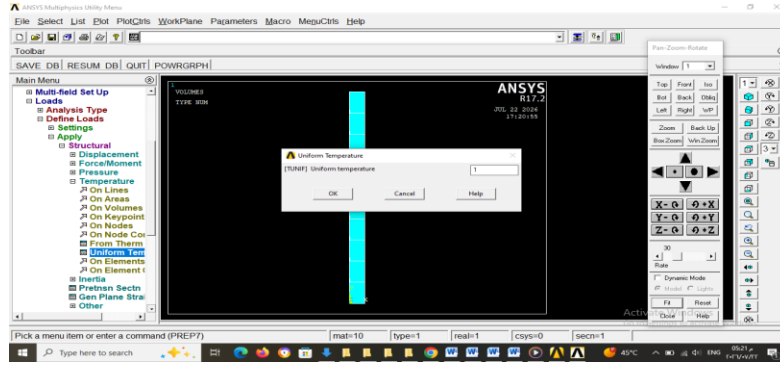
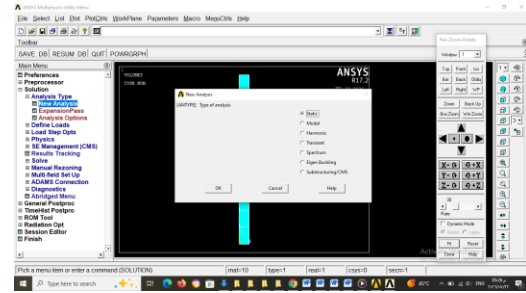
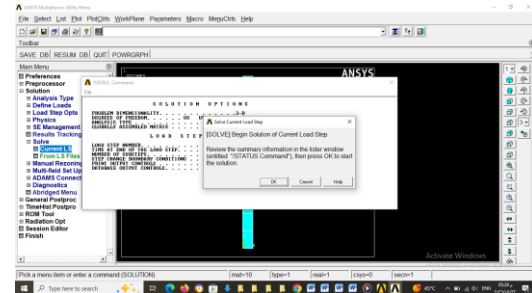


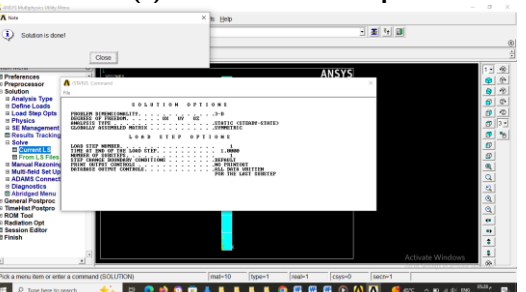
FIGURE 7 Step of Applying Thermal Load in the Thermal Buckling Problem



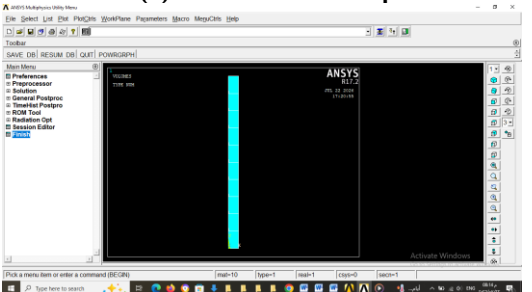
(a) Static Solution – Step1.



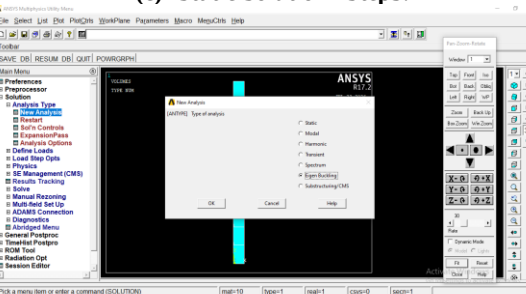
(b) Static Solution– Step2.



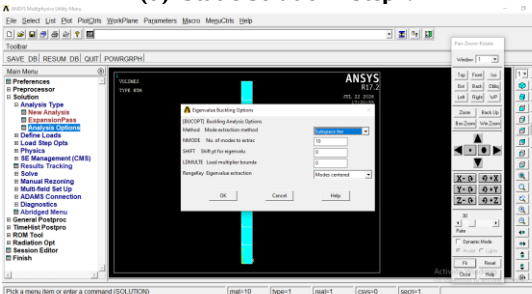
(c) Static Solution– Step3.



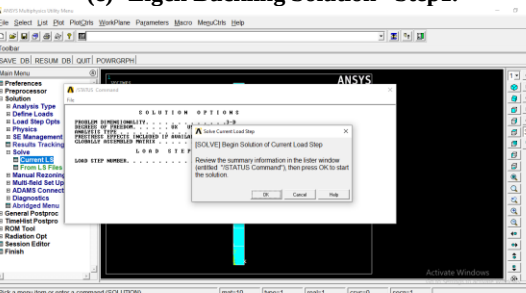
(d) Static Solution– Step4.



(e) Eigen Buckling Solution– Step1.



(f) Eigen Buckling Solution– Step2.



(g) Eigen Buckling Solution– Step3.



(h) Eigen Buckling Solution– Step4.

FIGURE 8 Steps of Solve the Thermal Buckling Problem

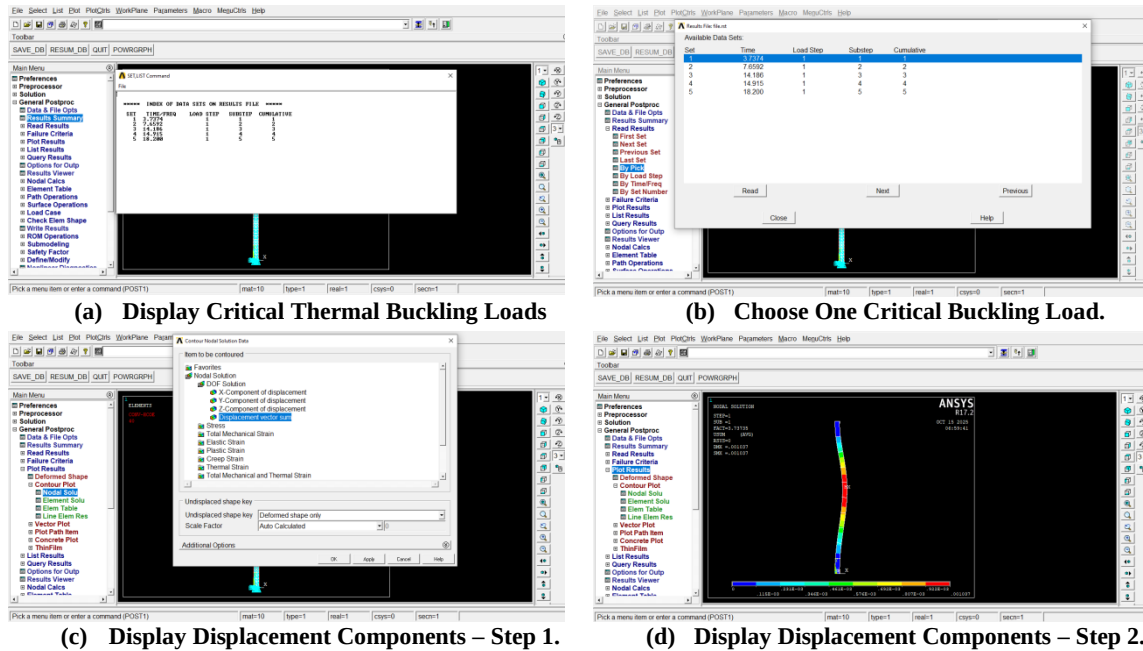


FIGURE 9 Steps of Displaying The Critical Thermal Buckling Loads and Displacement Components

5. CASES STUDIED

In this work, the effects of ratio of modulus ratio ($E_{Ratio} = E_{top}/E_{bottom}$), length-to-width ratio and supporting type (Fixed-Fixed and Simply supported-Fixed) on the critical heat buckling loads of FG-Fin are studied. The used modulus ratios are (0.2, 0.5, 0.75, 1, 2, 3 and 5) assuming copper is the top material, while the used L/W ratios are (0.25, 0.5, 0.75, 1, 1.5, 2 and 2.5) assuming constant side area of FG-fin. Finally, the content at the top of the FG-Fin is copper and its material properties are listed in Table (1), while the material properties of the material at the bottom of the FG-Fin are calculated depending on the value of modulus ratio.

Table 1 Material Properties of the Copper Used in this Work [27]

Property	Unit	Alumina (Al ₂ O ₃)	Copper (Cu)
Poisson Ratio	----	0.2	0.35
Modulus of Elasticity	GPa.	320	114
Thermal Conductivity	W/(m.K).	18	395
Thermal Expansion	1/K	6.5*10 ⁻⁶	16.5*10 ⁻⁶

6. RESULTS AND DISCUSSION

6.1 Distribution of Functionally Graded Material Properties

According to linear model (see Eq. (2)), Figure (10) shows the distribution of material properties (modulus of elasticity) for FGM assuming that the copper is the material of top surface (or top material). In Figure (10-a), the elastic modulus changes linearly with increasing the dimensionless length (y/L). Depending on modulus ratio ($E_{Ratio} = E_{top}/E_{bottom}$), three types of change in elastic modulus are shown. The first type appears when ($E_{Ratio} = 1$), there is no change in elastic modulus (i.e. constant elastic modulus), and this material is homogenous material. The second type appears when ($E_{Ratio} < 1$), the elastic modulus decreases with increasing the dimensionless length (y/L) as shown when ($E_{Ratio} = 0.2, 0.5$ and 0.75). The third one appears when ($E_{Ratio} > 1$), the elastic modulus increases with increasing the dimensionless length (y/L) as shown when ($E_{Ratio} = 2, 3$ and 5). While ratios of Poisson ratio is assumed constant and equal (0.35/0.2) as shown in Figure (10-b). Therefore, Poisson ratio is varied along the length of FG-fin. Using the same linear model, the thermal expansion coefficient is varied along the length of FG-Fins as shown in Figure (10-c).

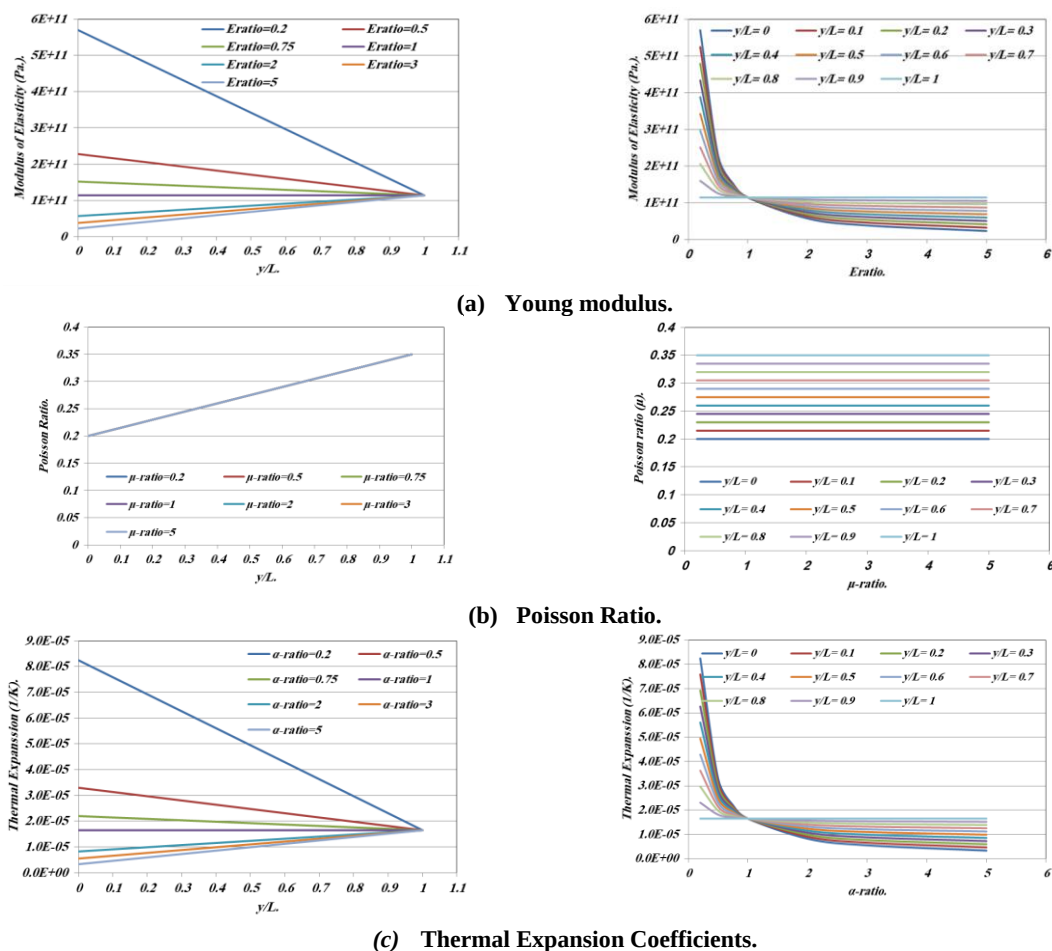


FIGURE 10 Variation of Elastic modulus Along the Length of Axial FG-Plate

6.2 ANSYS Model Validation

To verify the current work's authenticity, the results of the present work are compared with those obtained by Cavalcante et al. [24] assuming square homogenous plate with fixed four edges. The material properties of the homogenous materials are listed in Table (2). Table (3) shows a strong agreement between the results of the current study and those by Cavalcante et al. [24] and the highest percentage of absolute disparity is about (5.654%) when the length-to- thickness ratio is (40) and the material is Alumina (Al_2O_3).

Table 2 Material Properties Used by Cavalcante et al. [24]

Property	Unit	Alumina (Al_2O_3)	Ni
Poisson Ratio	----	0.25	0.3
Modulus of Elasticity	GPa.	393	199.5
Thermal Expansion	$1/^\circ\text{C}$	8.8×10^{-6}	13.3×10^{-6}

Table 3 The Comparison Between the Critical Buckling Loads Calculates by the ANSYS Software and that Obtained by Cavalcante et al. [24]

Length to Thickness Ratio	Length (m)	Thickness (m)	Present Work		Cavalcante et al. [24]		Discrepancy %	
			Al_2O_3	Ni	Al_2O_3	Ni	Al_2O_3	Ni
20	0.02	0.001	971.65	616.35	960	607	-1.214	-1.544
40	0.04	0.001	340.2	220	322	210	-5.654	-4.762
60	0.06	0.001	94.75	59.789	90	57	-5.277	-4.893
80	0.08	0.001	59.812	39.313	57	38	-4.933	-3.455
100	0.1	0.001	39.589	25.184	40	23	1.028	-9.496

6.3 Effect of Modulus and Length-to-Width Ratios

Figure (11) shows the essential thermal buckling load variation due to increase modulus ratio of fixed-fixed FG-fins at different length-to-width ratio. Generally, the first, second and third critical thermal buckling loads increase with increasing modulus ratio. This happens as a result of linear variation of modulus of elasticity and thermal expansion in additional to the length of the FG-Fin. When the length-to-width ratio is constant, the increase of modulus ratio (i.e. increase thermal expansion too (α -Ratio)) leads to increase the thermal expansion coefficient of the bottom material (α_{bottom}), therefore, the equivalent modulus of elasticity and equivalent thermal expansion coefficient increase too. In other side, the increase of length of FG-Fin causes decreasing of the critical thermal buckling load according to ($F_{\text{critical buckling load}} = \alpha_{\text{Equivalent}} * (DT) * L$) assuming $\alpha_{\text{Equivalent}}$ is constant when (α -Ratio) is constant. In other words, the modulus ratio (R-Ratio) and thermal expansion (α -Ratio) are proportional directly with thermal critical buckling at any length-to-width ratio. But, the thermal critical buckling is proportional inversely with the length-to-width ratio at any modulus ratio (R-Ratio) (and thermal expansion (α -Ratio)). The effect of variation material properties on the critical thermal buckling load is greater than that of length-to-width ratio and this fact is shown in Figures (12-14). Figures (12-14) show the increasing of first, second and third mode of critical heat buckling load respectively with rising both length-to-width ratio and modulus ratio. Also, the crucial thermal buckling load increases with increasing the mode number, but the value of this increasing depends on several parameters (theses parameters will discussed in next sections).

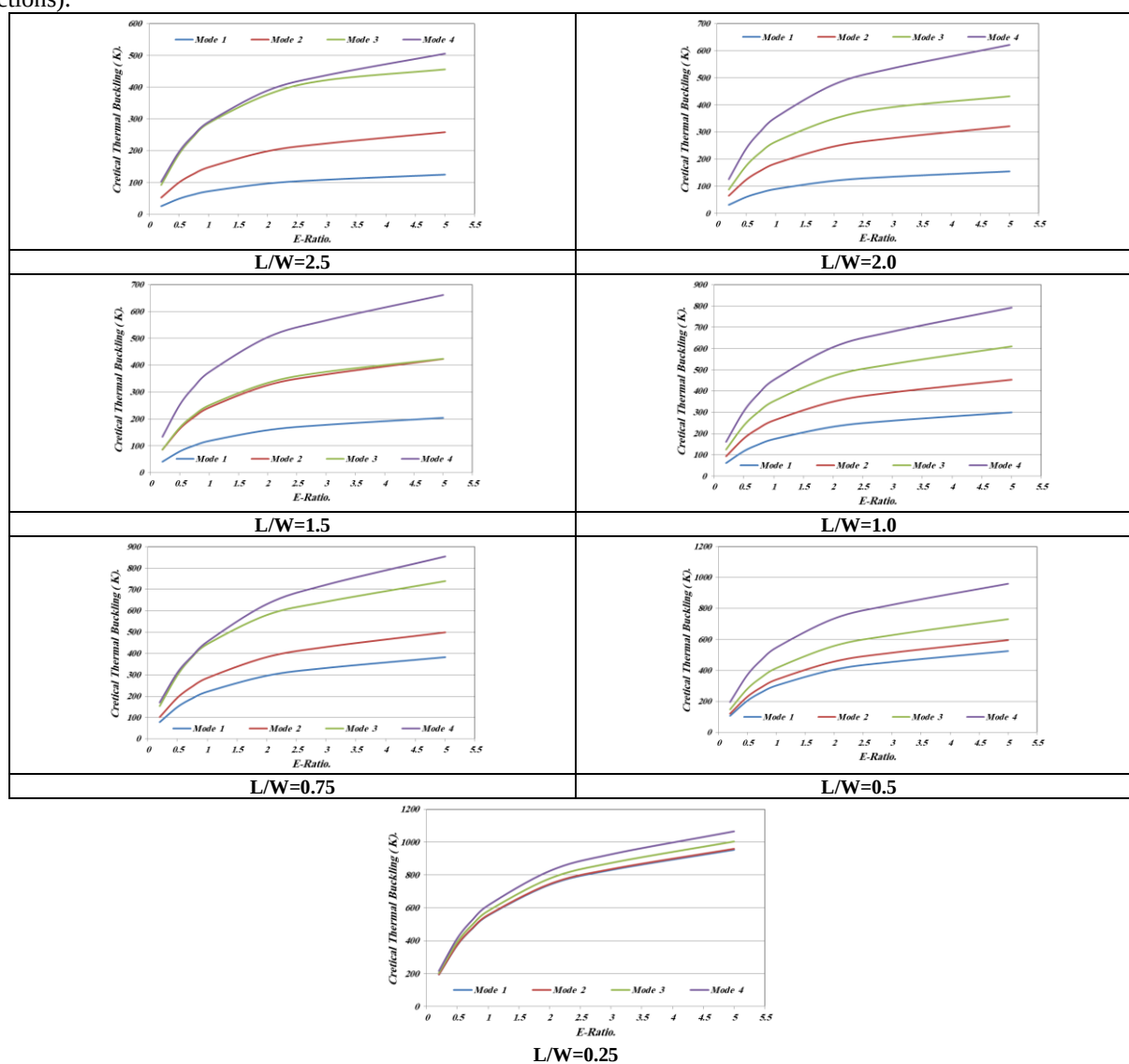
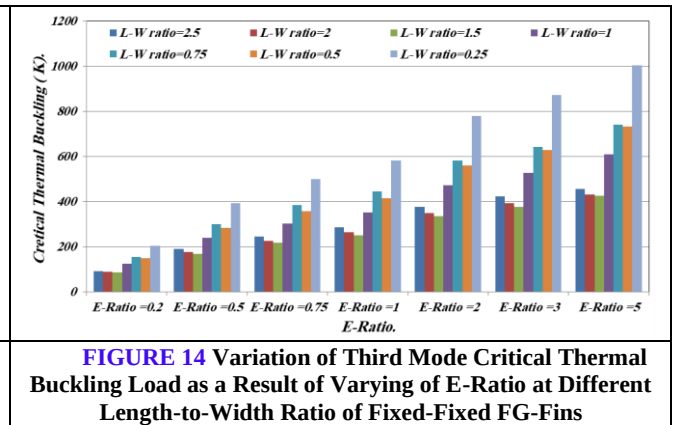
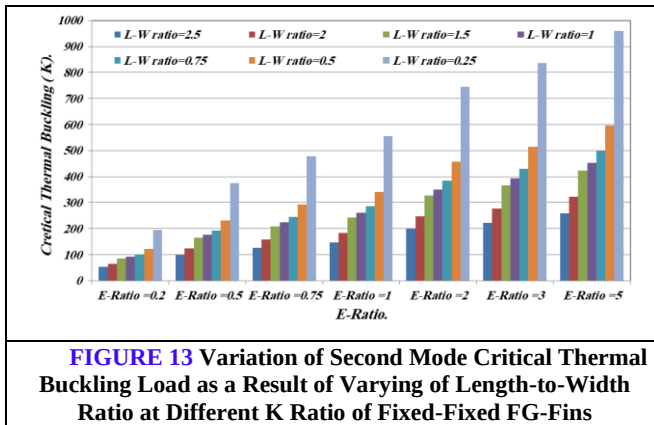
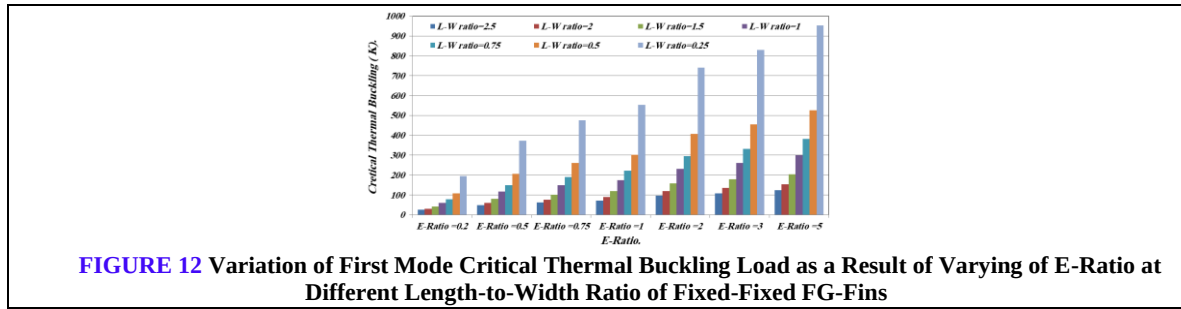


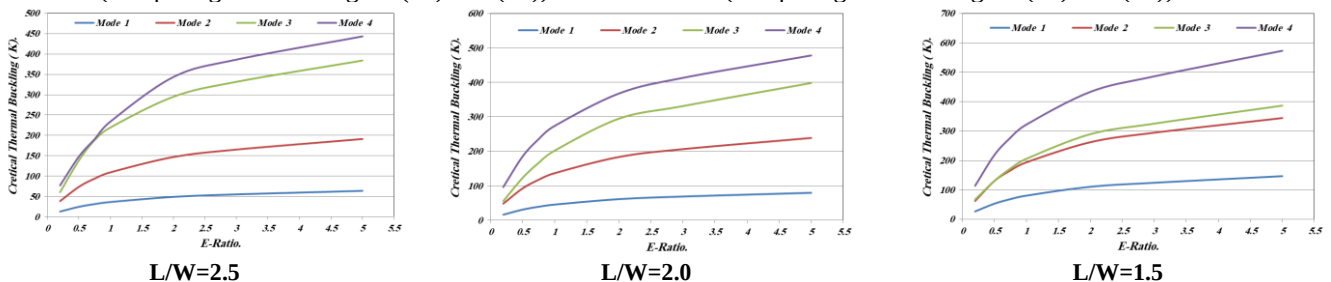
FIGURE 11 Variation of Critical Thermal Buckling Load as a Result of Varying of E-Ratio at Different Length / Width Ratio of Fixed-Fixed FG-Fins

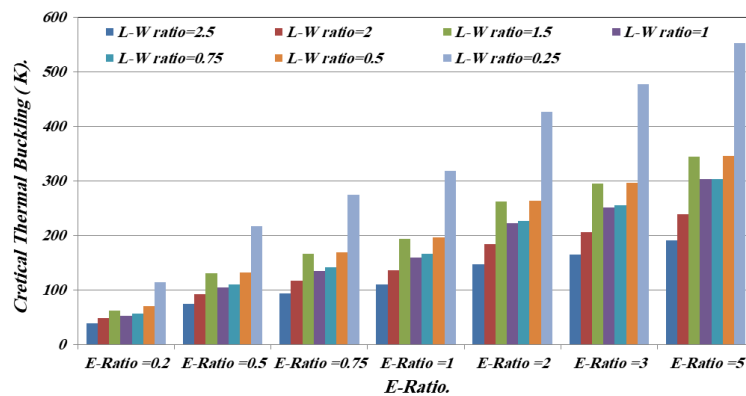
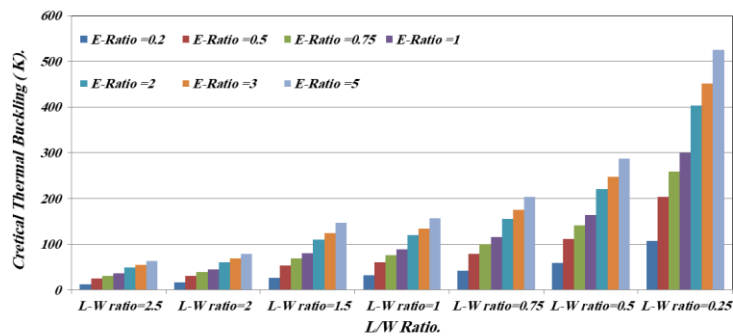
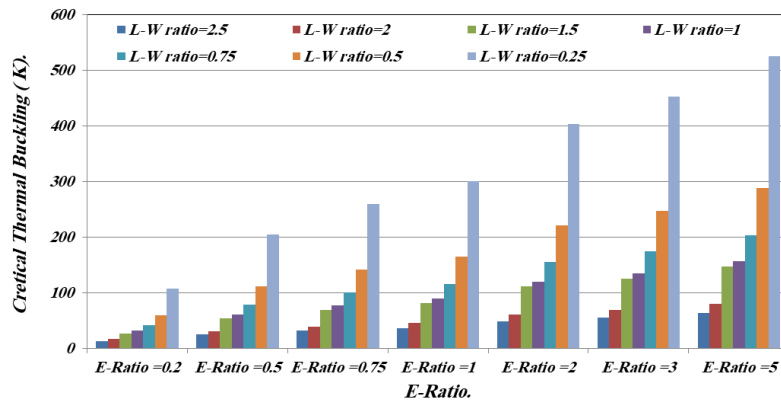
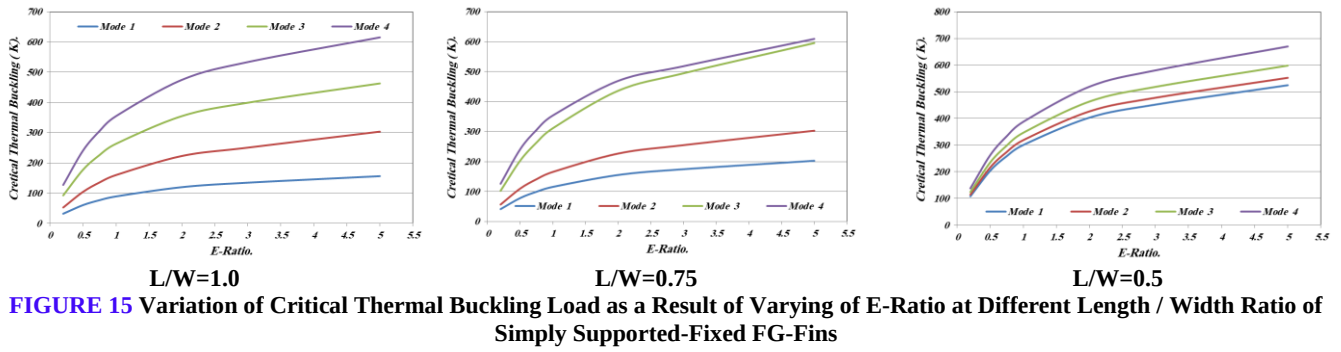


For simply support-fixed FG-fins, Figure (15) shows the variation of critical thermal buckling load due to increase modulus ratio at different length-to-width ratio. Similar to fixed -fixed FG-fins, the first, second and third crucial thermal buckling load s increase with rising modulus ratio for the same reasons discussed previously. Also, the modulus ratio (R-Ratio) and thermal expansion (α -Ratio) are proportional directly with thermal critical buckling at any length-to-width ratio. But the thermal critical buckling is proportional inversely with the length-to-width ratio at any modulus ratio (R-Ratio) (and thermal expansion (α -Ratio))). Also, the effect of variation material properties on the critical thermal buckling load is greater than that of length-to-width ratio (see Figures (16-18)). Figures (16-18) show the increasing of first mode of critical thermal buckling load with increasing both length-to-width ratio and modulus ratio. Also, the second mode of critical thermal buckling load increases with increasing both length-to-width ratio and modulus ratio. Finally, the third mode of critical thermal buckling load increases randomly with increasing both length-to-width ratio and modulus ratio. Also, the crucial heat buckling load increases with increasing the mode number, but, the value of this increasing depends on several parameters.

6.4 Effect of Supporting Type

By comparing between Figure (12) and (16), it can be noted that the crucial heat buckling load of fixed-fixed FG-fin is greater than that of simply support-fixed FG-fin because of type of support. The same behavior can see for second model (comparing between Figure (13) and (17)) and third mode (comparing between Figure (14) and (18)).





6.5 Shape Mode

By comparing between Figure (14) and (18), anyone can see irregular variation in the third mode of the critical thermal buckling load. This irregular variation happens because of changing the shape mode of FG-fins as illustrated in Figures (19 and 20). From Figures (19 and 20), the first shape modes for FG-fins when ($L/W=0.25$) and for the two supporting types at different modulus ratio (E Ratio=0.2 ,0.75 ,1 and 5) are similar. Also, the same fact is shown for the second shape modes for FG-fins. But, there is a slight difference in the third shape modes for FG-fins. The difference in the third shape modes for FG-fins appears sharply when ($L/W=0.75$) as shown in Figures (21 and 22) and when ($L/W=2$) as shown in Figures (23 and 24). From Figures (19-24), the first, second and third shape modes change due to change the length-to-width ratio, therefore, we can concluded that the length-to-width ratio is very important parameters affected on the shape mode creation.

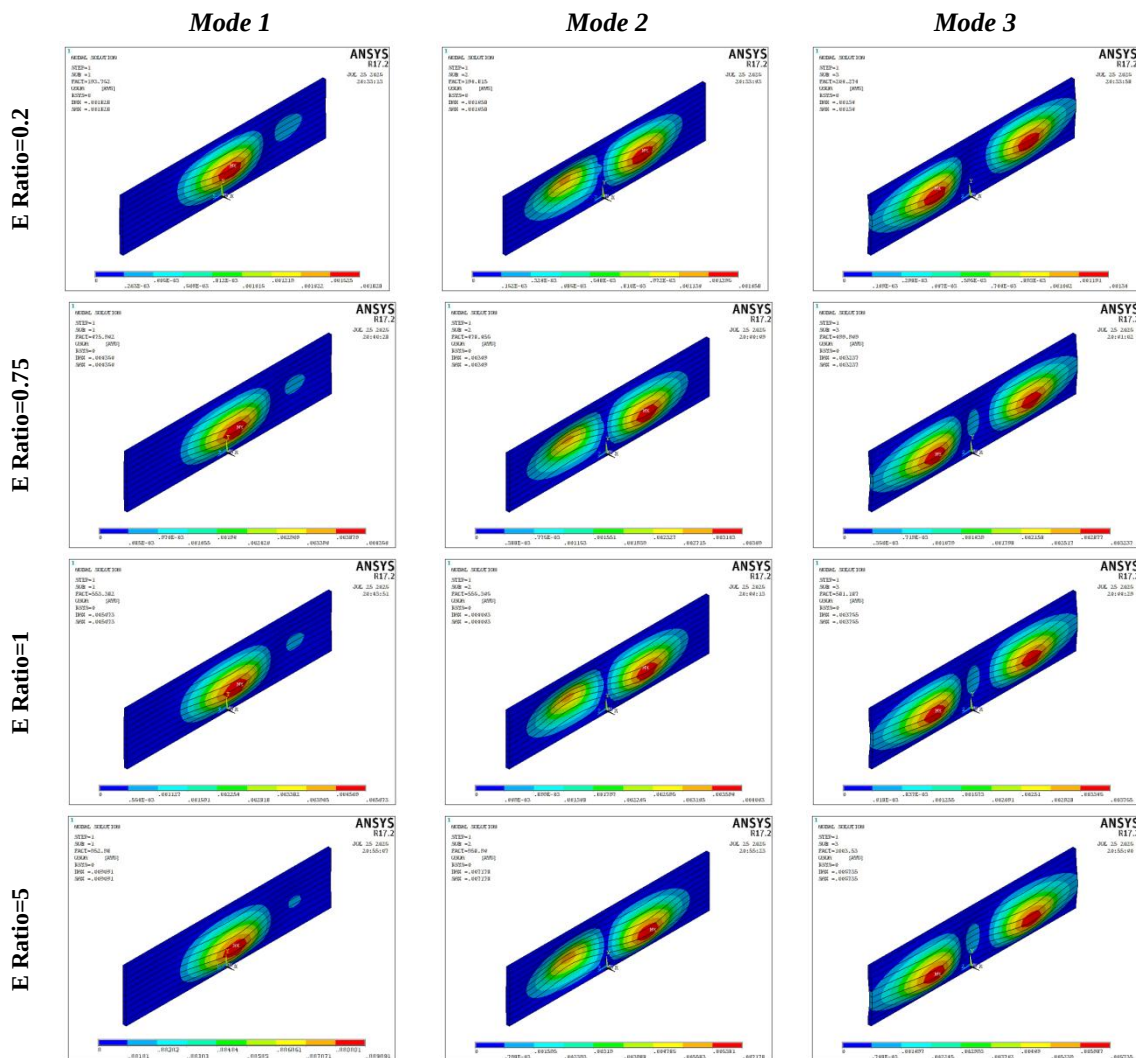


FIGURE 19 The First Three Shape Modes of Fixed-Fixed FG-Fins at Different Modulus Ratio and When $L/W=0.25$

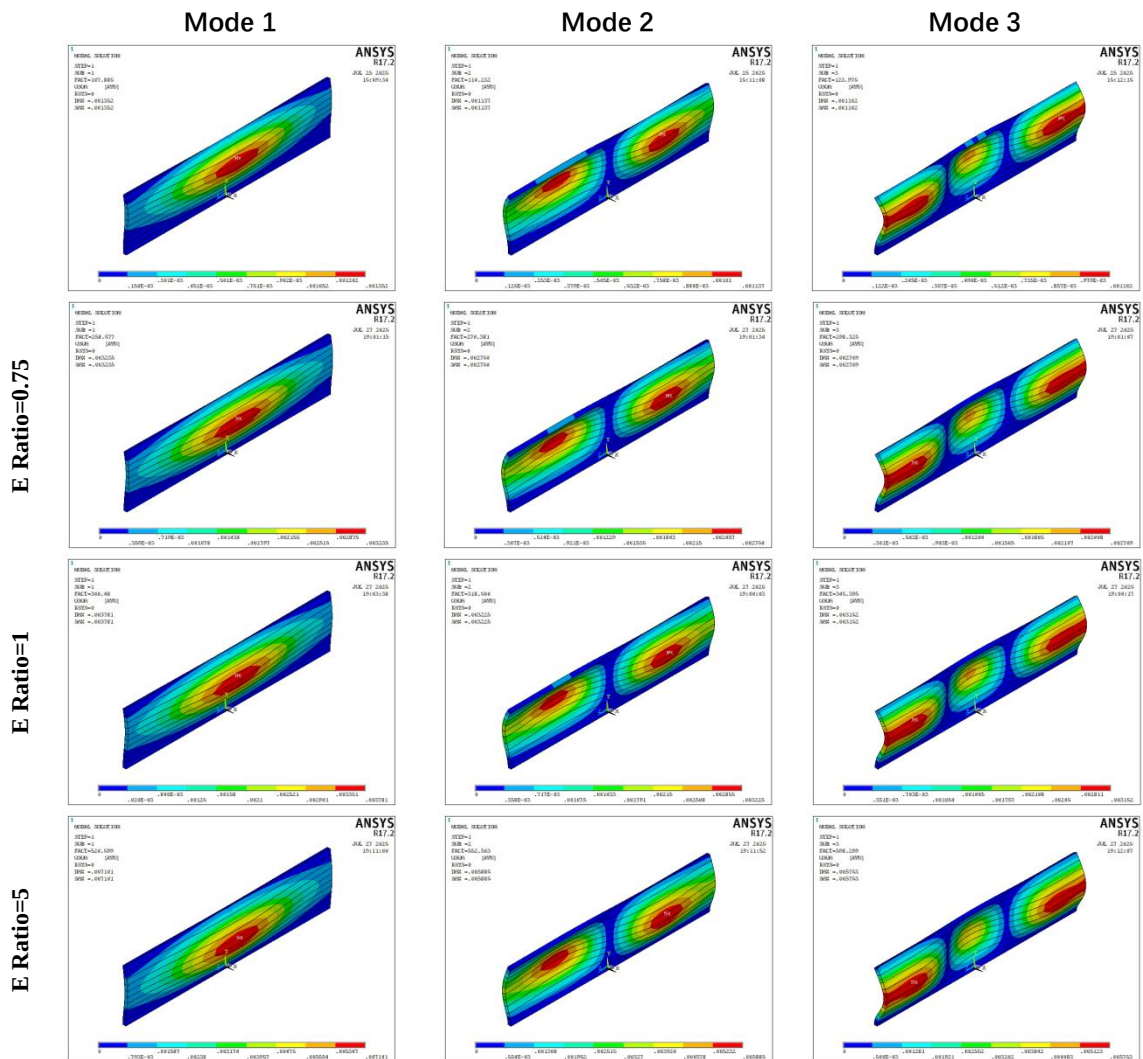
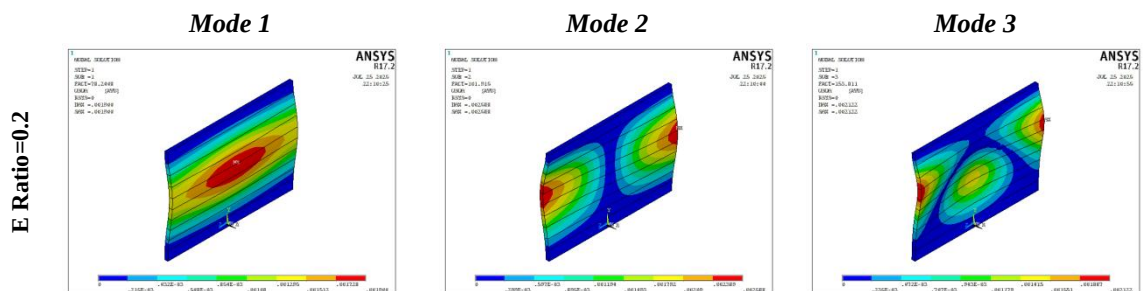


FIGURE 20 The First Three Shape Modes of Simply Support-Fixed FG-Fins at Different Modulus Ratio and When $L/W=0.25$



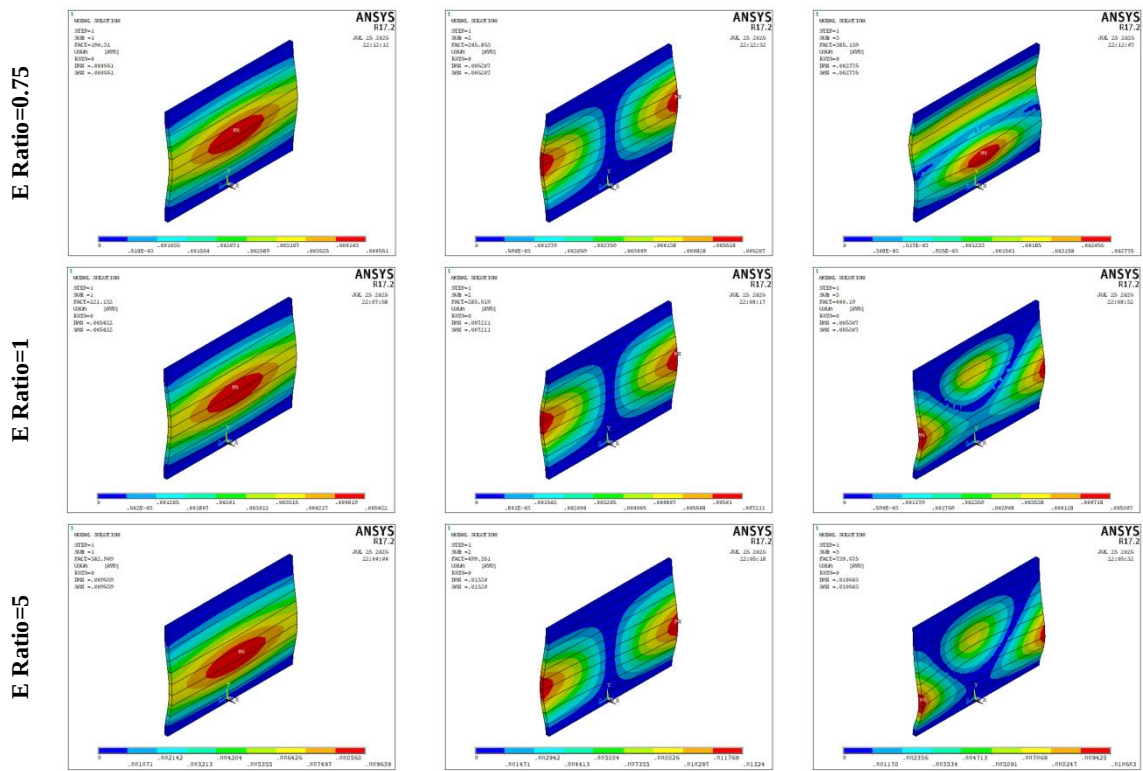
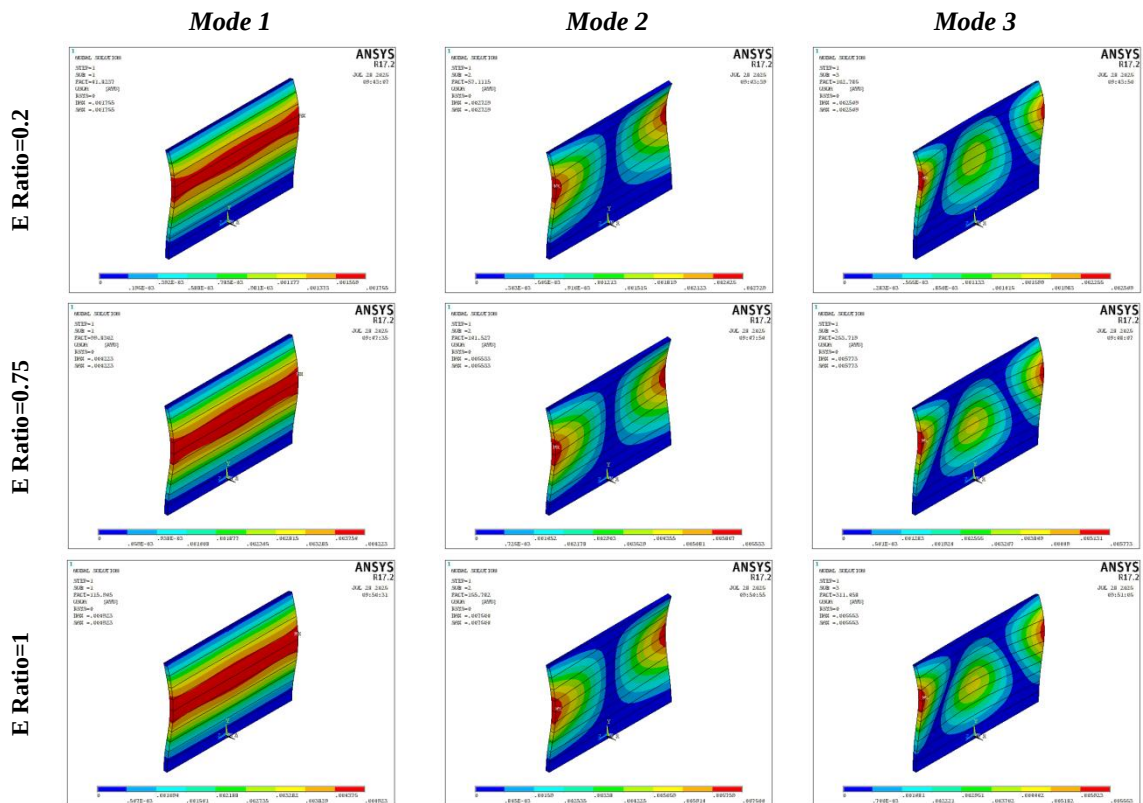


FIGURE 21 The First Three Shape Modes of Fixed-Fixed FG-Fins at Different Modulus Ratio and When $L/W=0.75$



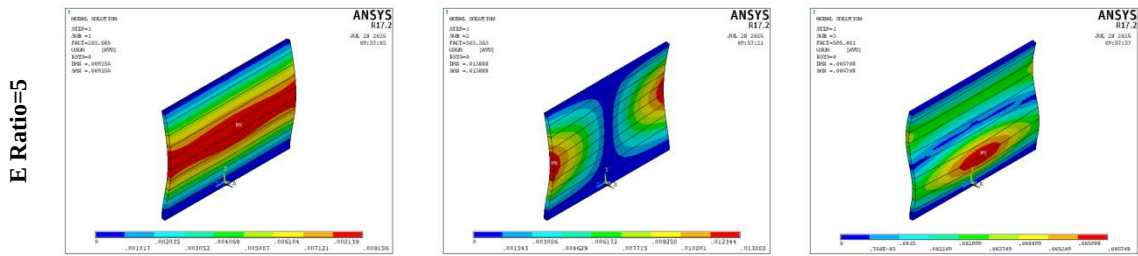


FIGURE 22 The First Three Shape Modes of Simply Support-Fixed FG-Fins at Different Modulus Ratio and When $L/W=0.75$

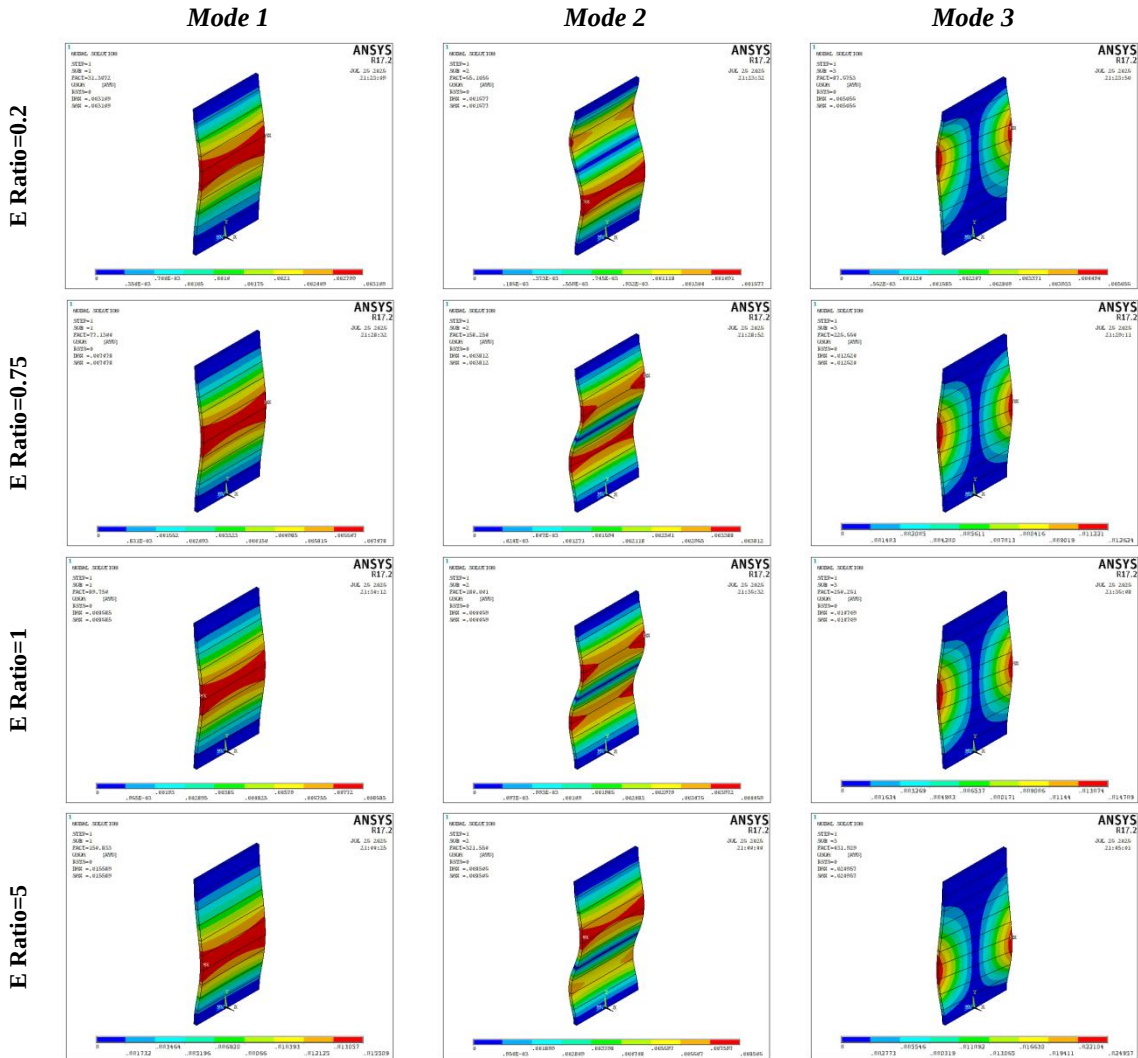


FIGURE 23 The First Three Shape Modes of Fixed-Fixed FG-Fins at Different Modulus Ratio and When $L/W=2$

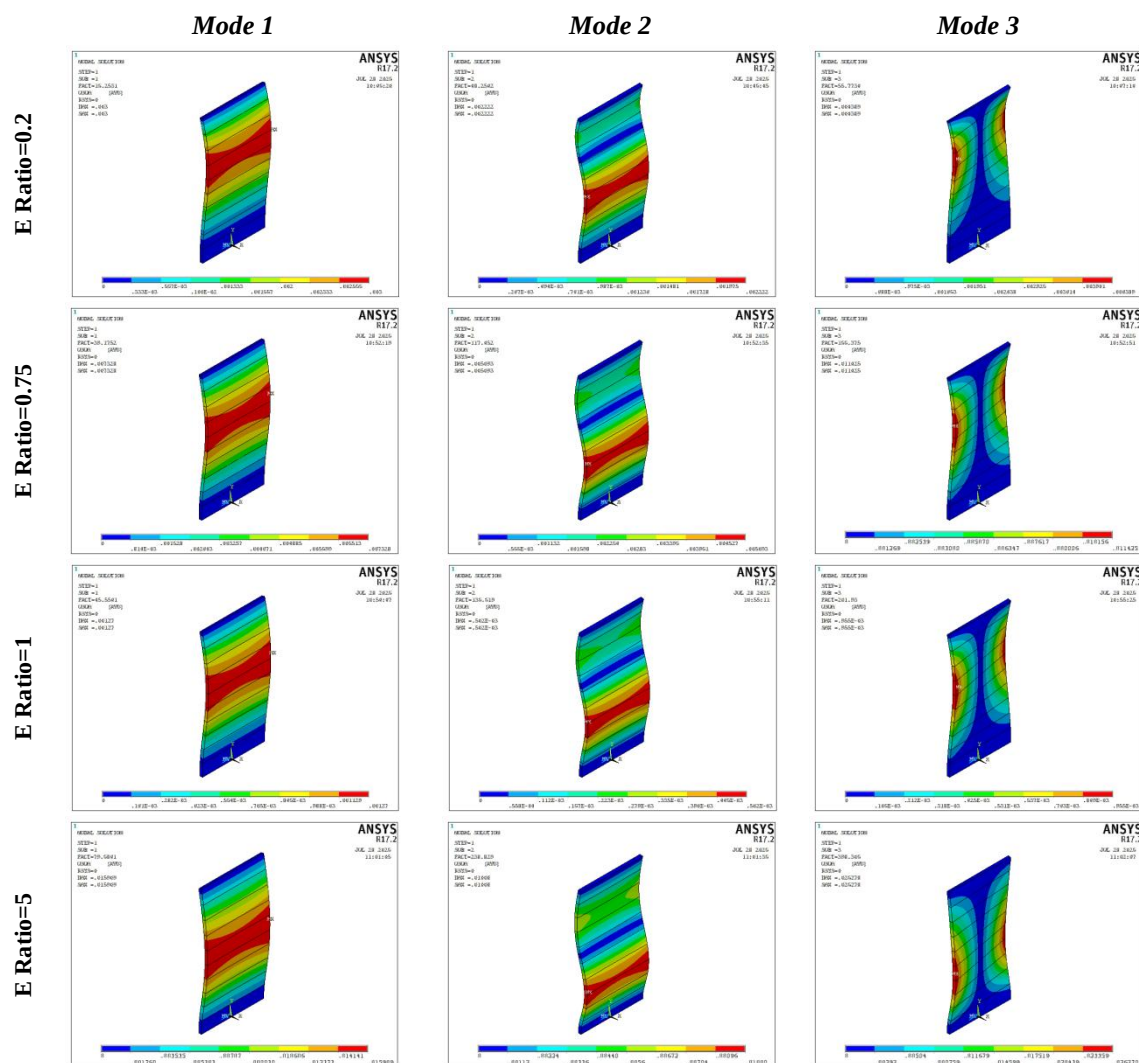


FIGURE 24 The First Three Shape Modes of Simply Support-Fixed FG-Fins at Different Modulus Ratio and When $L/W=2$

7. Conclusions and Future Work

In this study, the effects of modulus ratio (E Ratio) (and thermal expansion ratio (α Ratio) (E Ratio= α Ratio)) (E Ratio=0.2, 0.5, 0.75, 1, 2, 3 and 5) and length-to-width ratio ($L/W=0.25, 0.5, 0.75, 1, 1.5, 2$ and 2.5) on the critical heat buckling load of FG-fins with two types of supporting (Fixed-Fixed and Simply Support-Fixed). This work used linear buckling to describe the variation of material properties along the length of FG-Fin and assumed constant temperature to calculate the critical thermal buckling load. From the results of this work, the following point can be concluded:

1-For the two types of supports, the crucial thermal buckling load rises as the length-to-width ratio decreases of FG-fin at the same modulus ratio (E Ratio).

2-For the two types of supports, the critical thermal buckling load increases with increasing modulus ratio (E Ratio) of FG-fin at the same length-to-width ratio.

3-The crucial heat buckling loads of the simply supported-fixed FG-fin is smaller than that of fixed -fixed FG-fin at the same modulus ratio (E Ratio), thermal expansion ratio (α Ratio), ratio of length to breadth and mode number.

4-Due to change the shape mode, irregular critical thermal buckling loads are found at the third mode.

5-The length-to-width ratio is an important parameter affected on the shape mode of FG fin. The shape mode effects on the issue with heat transfer and failure behavior of the FG-fin.

In future work, the following points can be achieved to improve this work:

1-The analytical method can be improved to calculate the critical thermal buckling loads of FG Fin.

2-The numerical method can be improved to calculate the critical thermal buckling loads of FG-Fin under different temperature profiles.

3-The structural boundary conditions can change to be (fixed- fixed-fixed fixed) or (simply –simply-simply –simply) conditions.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest

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